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ISOGEOMETRIC FINITE ELEMENT ANALYSIS OF SINGLE-PHASE DARCY FLOW IN POROUS MEDIA

Yared W. Bekele[1], Arne M. Kvarving[2], Steinar Nordal[3], Trond Kvamsdal[4] and Gustav Grimstad[5]

[1,3,4,5] Norwegian University of Science and Technology, yared.bekele@ntnu.no, steinar.nordal@ntnu.no, trond.kvamsdal@sintef.no, gustav.grimstad@ntnu.no
[2] SINTEF ICT, arne.morten.kvarving@sintef.no

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Isogeometric finite element analysis is a relatively new computational method that directly employs complex non-uniform rational B-splines (NURBS) in the finite element method. Since it was first introduced to fill the gap between computer aided design and finite element analysis, [1], isogeometric finite element analysis has shown several advantages over the conventional finite element method. The perfomance of isogeometric analysis in porous media and its advantages specifically in poroelasticity were shown in [2]; the study showed that, because of the higher-order continuity of the spline basis functions, a better continuity of pressure gradients is observed.

In this study, the isogeometric finite element method is applied to a single-phase Darcy flow in porous media. The problem is defined as:

$$
\begin{aligned}
\mathbf{w} = -\frac{\mathbf{k}}{\rho^w g}\left(\nabla p^w - \rho^w \mathbf{b}\right) &\quad \text{in } \Omega \\
\nabla \cdot \mathbf{w} = 0 &\quad \text{in } \Omega \\
\rho^w \left(\mathbf{w} \cdot \mathbf{n}\right) = q^w &\quad \text{on } \Gamma_N \\
p^w = \hat{p}^w &\quad \text{on } \Gamma_D
\end{aligned}
$$

where $\mathbf{w} : \Omega \to \Re^n$ is the Darcy velocity vector, $\mathbf{k}$ is the permeability matrix, ρ^w is the fluid density, g is the acceleration due to gravity, $p^w : \Omega \to \Re^n$ is the pore fluid pressure and $\mathbf{b}$ are the body forces. The second equation indicates steady-state flow. The last two equations define the Dirichlet and Neumann boundary conditions, respectively, for the problem with $\Gamma = \Gamma_N \cup \Gamma_D$.

The variational formulation and finite element discretization of the problem is performed for the problem defined above. The single variable formulation focuses on the pressure as a primary unknown variable.

Weak formulation and spatial discretization of the weak from, with pressures as the main variables, results in the matrix equation:

$$\mathbf{B}\bar{\mathbf{p}}^w = \mathbf{f}^p$$

where:

$$\begin{aligned}\mathbf{B} &= \int_\Omega (\nabla \mathbf{N}_p)^T \frac{\mathbf{k}}{\rho^w g} (\nabla \mathbf{N}_p)\, d\Omega \\ \mathbf{f}^p &= \int_\Omega (\nabla \mathbf{N}_p)^T \frac{\mathbf{k}}{g} \mathbf{b} d\Omega - \int_{\Gamma_q} \mathbf{N}_p^T \frac{q^w}{\rho^w} d\Gamma\end{aligned}$$

In the above equations, $\mathbf{N}_p$ are the shape functions for the unknown pressure field $\bar{\mathbf{p}}^w$. Numerical implementation of the linearized equations and verification is performed in an open-source software called IFEM.

The implementation is verified by simulating groundwater flow in a homogeneous aquifer. An isotropic porous medium is first assumed. The model is then used to simulate groundwater flow in an anisotropic porous medium, in order to consider the anisotropy of permeability. The results are compared with the conventional finite element method.

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