11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

UPPER BOUND LIMIT ANALYSIS OF 3D PROBLEMS USING DISCONTINUITY LAYOUT OPTIMIZATION

***Samuel J. Hawksbee[1], Matthew Gilbert[1] and Colin C. Smith[1]**

[1] Computational Mechanics and Design Research Group, Dept Civil & Structural Engineering,
University of Sheffield, Mappin Street, Sheffield S1 3JD, UK.
Web: http://cmd.shef.ac.uk.
Email: {s.hawksbee, m.gilbert, c.c.smith}@sheffield.ac.uk.

Key words: *Discontinuity Layout Optimization; Limit Analysis; Plasticity.*

Limit analysis provides a powerful and direct means of evaluating the safety of a solid body or structure. However, a barrier to employing the upper bound form of limit analysis has always been identifying an appropriate collapse mechanism, required in order to obtain an accurate, and sufficiently 'safe', engineering solution. Automating the process of identifying an appropriate collapse mechanism has therefore been a goal of researchers for many decades. Relatively recently the discontinuity layout optimization (DLO) procedure has been proposed as an efficient and systematic means of doing this [1]. With DLO the upper bound theorem of limit analysis can be framed purely in terms of discontinuities, and for plane-strain problems linear programming can be used to obtain solutions. An important advantage of DLO is its ability to handle singularities, without the need for 'tailoring' of the numerical discretization or mesh refinement. Recently, Hawksbee et. al. [2] demonstrated that DLO could also be applied to three-dimensional problems. The differences between the plane-strain and three-dimensional formulations of DLO are illustrated by Table 1. The three-dimensional formulation has been used to obtain good results for a number of simple benchmark problems, demonstrating its potential.

Table 1: Comparison of plane-strain and three-dimensional DLO formulations

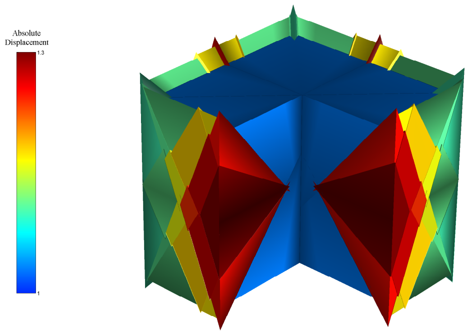
	Plane-strain [1]	Three-dimensional [2]
d.o.f. (per discontinuity):	2	3
discontinuities:	line segments	planar polygons
compatibility enforced at:	nodes	edges
optimization problem type:	linear programming	second order cone programming

Table 2 and Figure shows example results for two examples: compression of a rectan-

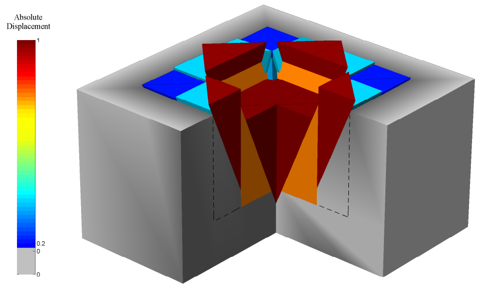
Table 2: Example results (after [2])

Problem	Total no. discontinuities	Solution	Difference (%)
compression of a block	7, 704	2.319	0.60†
compression of a block	23, 100	2.307	0.072†
compression of a block	133, 884	2.304	0.043†
bearing capacity N_c	157	6.521	7.8‡
bearing capacity N_c	917,472	6.102	0.84‡
bearing capacity N_c	5,114,220	6.044⋆	-0.12‡

† Compared with best published upper bound [4] ; ‡ compared with best published upper bound [3]; ⋆ new solution.



(a)



(b)

Figure 1: Representative failure mechanisms (after [2]): (a) compression of a block (7, 704 discontinuities); (b) punch indentation (157 discontinuities, dashed lines indicate extent of domain modelled)

gular block of Tresca material and bearing capacity of a square punch bearing onto a Tresca material. Work to improve the computational efficiency of the method is ongoing, particularly focussing on decompositional methods.

REFERENCES

[1] C.C. Smith and M. Gilbert. Application of discontinuities layout optimization to plane plasticity problems. *Proc. R. Soc. A*, Vol. **463**, 2461–2484, 2007.

[2] S. Hawksbee, C.C. Smith and M. Gilbert. Application of discontinuity layout optimization to three-dimensional plasticity problems. *Proc. R. Soc. A*, Vol. **469**, 2013.

[3] Vicente da Silva, M. V. A. N. Antão. Upper bound limit analysis with a parallel mixed finite element formulation. *Int. J. Solids Struct.*, Vol . **45**, 5788-5804, 2008.

[4] C. Martin and A. Makrodimopoulos. Finite-element limit analysis of Mohr-Coulomb materials in 3D using semidefinite programming. *J. Eng. Mech.*, Vol. **134**, 339-347, 2008.