

A COMPUTATIONAL INVESTIGATION OF HARDENING RELATIONS FOR GRADIENT SINGLE-CRYSTAL PLASTICITY

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Key words: *single-crystal plasticity, gradient plasticity, self-hardening, latent-hardening*

A variety of hardening relations have been proposed in the context of single-crystal plasticity. A popular and widely used relation is one in which the evolution of slip resistances, in the form of a system of ordinary differential equations, is expressed as a function of the resistances and the slip rates (see for example [5, 4]). Such relations have been extended to strain-gradient theories of single-crystal plasticity essentially by replacing the slip-rates with a scalar measure of slip-rates and their gradients [2].

In recent work [3], a rate-independent, thermodynamically consistent theory of strain-gradient single-crystal plasticity has been developed. The theory is based on an earlier treatment of the viscoplastic problem in [1], in which microscopic scalar and vector stresses, which are power-conjugate to slip-rates and their gradients, are introduced. A development of the rate-independent counterpart, which incorporates a yield function and generalized Mises-Hill flow relation, has been presented in [6].

A novel feature in [3] is the presentation of a new hardening relation in which both self- and latent-hardening are accounted for. Rather than taking the form of an evolution equation, the hardening relation appearing in the yield condition is a function of accumulated generalized slip, with self- and latent-hardening terms appearing additively. The full problem is cast in weak form as a combination of a variational inequality for the flow relation together with microscopic force balance, and an equation for macroscopic equilibrium. It is shown that the time-discrete form of the problem can be formulated as a minimization problem.

The above developments have taken place in the context of small deformations. The

aim of this presentation is, first, to extend the formulation and analysis into the large-deformation range. As in the small-strain case the variational setting takes the form of a variational inequality and equation, and it is again possible to obtain a minimization problem corresponding to the time-discrete approximation.

The second objective is to present the results of a comparative study of hardening relations, through a series of computations for both the conventional and strain-gradient problems. The computations highlight the simpler implementation of the new hardening relation while also illustrating the nature of self- and latent-hardening.

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