## COMPARISON AND COMBINATION OF POINT-BASED AND SEGMENT-BASED ISOGEOMETRIC CONTACT FORMULATIONS

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Formulation of isogeometric finite elements [1] has recently received a great deal of attention. One decisive difference of isogeometric finite elements (FE) based on NURBS functions compared to standard FE based on Lagrange Polynomials is higher inter-element continuity. This is a promising devise to model contact problems, as all issues associated with kinks between elements are naturally avoided. Particular in cases with large sliding this has the potential to be an attractive feature, as already mentioned by Hughes et al. [1].



Figure 1: Hertz problem: pre-integration of contact surface, linear Lagrange Multiplier field

In order to perform contact analysis, a non-penetration condition is added to the total energy. Several methods are available, like the Lagrange Multiplier Method, the Penalty Method, the Augmented Lagrange Method and many more. With the Lagrange Multiplier Method, the contact condition can be enforced exactly. This includes the introduction of additional degrees of freedom, increasing the size of the global system of equations. Choosing a spatial discretization for the contact condition is the next step. One of the first methods for non-matching meshes was the so-called node-to-segment (NTS) approach, which enforces the non-penetration condition pointwise. Unfortunately this method suffers from numerical instabilities using the standard FE basis functions and it does not pass the patch test. Transferring the NTS method to isogeometric analysis resolves the problem of numerical instabilities by using the smooth NURBS basis. Moreover, the new basis poses the question where to collocate the contact integral, because in a NURBS representation the nodes (control points) are not necessarily part of the current geometry. Consequently alternative collocation points for enforcing the contact constraints have to be defined on the contact surface which yields a point to segment formulation (PTS), see [2]. One drawback still remains, it does not pass the contact patchest.

The patchtest can be passed if the contact integral is not collocated but numerically integrated in the framework of a segment-to-segment (STS) contact formulation, see [3]. The main differences of both methods are the number, location and weight of the evaluation points and the underlying Lagrange Multiplier field. However, the discrete contact equations for calculating the contribution to the global stiffness matrix are the same in both methods.

A third method is introduced here, which combines features of the PTS and the STS approach. A Lagrange Multiplier field is introduced as for the STS approach, but it is collocated at Greville or Botella points like in the PTS algorithm. Weights are assigned to these points which depend on the corresponding parts of the integration segments. An implementation of this method yields numerical results which are of the same accuracy as for the PTS algorithm (see Figure 1), but still the patchtest cannot be passed. One decisive advantage is that the Lagrange Multipliers correspond to contact stresses and not, as for the PTS method, to contact forces. This facilitates a direct and consistent stress recovery.

In the presentation, the three different approaches are compared with respect to implementation, results, stability and computational time.

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