11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A HYBRID-TREFFTZ PERFORATED ELEMENT FEATURING NODES ON THE HOLE BOUNDARY

C. Hennuyer[1,2], N. Leconte[2], B. Langrand[1], E. Markiewicz[2]

[1] ONERA - The French Aerospace Lab, F-59045 Lille, France
[2] LAMIH, UMR CNRS 8201, UVHC, F-59313 Valenciennes Cedex 9, France

Key words: *Hybrid-Trefftz, Super-Finite Element, Riveted joint modelling, Rivet/Perforated plate Interaction.*

Finite Element analysis of airframe under High Velocity Impacts hardly succeeds in representing the failure of the structure when it occurs in riveted joint areas [1]. The most advanced connector models are able to describe the non-linear behaviour of the rivet until rupture [2], but, the structural embrittlement caused by holes in the sheets (riveting process) is not introduced in the standard shell FE to which rivets macro-elements are connected [3]. The research aims to enhance the modelling of riveted joints in structural computations by developing "super-elements", which take into account the stress concentrations due to holes, and, the interaction between both the rivet and the sheet macro-models. The formulation of a 16-node Hybrid-Trefftz (HT) perforated super-FE featuring 8 nodes that can be loaded on the hole boundary is thus proposed in this paper.

The new perforated FE (Fig. 1) is a generalization of the 8-node HT FE developed by Leconte *et al.* [4] that does not features nodes on the hole boundary. Its formulation is based on a HT displacement variational principle. In the new formulation, the hole boundary (denoted by S_p) is now taken into account in the principle (1). The super-FE is supposed to be completely surrounded by conventional FE, and, is formulated so that the displacement compatibility is ensured only on the external boundary (denoted by S_i) and not on the internal boundary, $\widetilde{\mathbf{u}} = \widetilde{\mathbf{N}}\widetilde{\mathbf{q}}_{ext}$ (otherwise, it would correspond to the case of an inclusion [5]). The homogeneous solutions $\mathbf{u}^h$ and $\mathbf{T}^h$ (traction vector) are expressed by $\mathbf{u}^h = \mathbf{Ud}$ and $\mathbf{T}^h = \mathbf{Md}$, where $\mathbf{d}$ is the vector of external and internal nodal displacements, $\mathbf{U}$ and $\mathbf{M}$ are the shape functions calculated from complex variable methods [6]. Loads applied on the hole boundary are taken into account in the FE formulation by a particular solution, $(\mathbf{u}^p, \mathbf{T}^p)$, which is added to the homogeneous one. The particular solution considered here enables to apply concentrated forces on the nodes of the hole boundary [7]. The interaction with the rivet model can then be established.

$$\Pi_{\mathrm{HT}}\left[\mathbf{d}, \widetilde{\mathbf{q}}_{ext}\right] = \frac{1}{2}\mathbf{d}^t\mathbf{H}\mathbf{d} + \mathbf{d}^t\mathbf{L}\widetilde{\mathbf{q}}_{ext} + \mathbf{d}^t\mathbf{r}^c + \widetilde{\mathbf{q}}_{ext}^t\mathbf{r}^d \quad (1)$$

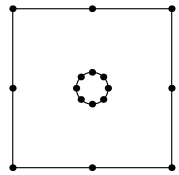

Figure 1: A 16-node HT super-FE featuring 8 nodes for the hole boundary.

with $\mathbf{H} = \int_{S_p} \mathbf{M}^t\mathbf{U}\ dS - \int_{S_i} \mathbf{M}^t\mathbf{U}\ dS$, $\mathbf{L} = \int_{S_i} \mathbf{M}^t\widetilde{\mathbf{N}}\ dS$, $\mathbf{r}^d = \int_{S_i} \widetilde{\mathbf{N}}^t\mathbf{T}^p\ dS$, and $\mathbf{r}^c = \frac{1}{2}\left[\int_{S_p} \mathbf{M}^t\mathbf{u}^p\ dS + \int_{S_p} \mathbf{U}^t\mathbf{T}^p\ dS - \int_{S_i} \mathbf{M}^t\mathbf{u}^p\ dS - \int_{S_i} \mathbf{U}^t\mathbf{T}^p\ dS\right]$.

If the minimization of the Π_{HT} functional was performed with respect to $\mathbf{d}$ and $\widetilde{\mathbf{q}}_{ext}$, it would lead to a system where the internal degrees of freedom would be eliminated from the formulation. To keep $\mathbf{q}_{int}$ into the element formulation, it is necessary to decompose $\mathbf{d} = [\mathbf{q}_{ext}^t, \mathbf{q}_{int}^t]$ in order to express Π_{HT} as a function of $\mathbf{q}_{ext}$, $\mathbf{q}_{int}$ and $\widetilde{\mathbf{q}}_{ext}$. The stiffness matrix $\mathbf{K}$ of the new FE is obtained by taking stationary conditions with respect to $\mathbf{q}_{ext}$ and $\mathbf{q} = [\widetilde{\mathbf{q}}_{ext}^t, \mathbf{q}_{int}^t]$, successively. The expected system $\mathbf{Kq} = \mathbf{f}$ is finally obtained: the compatibility condition is enforced on the external boundary only, and the FE features nodes on its hole boundary. The hole boundary loading is performed through the force vector $\mathbf{f}$ which is composed of $(\mathbf{u}^p, \mathbf{T}^p)$. The new perforated super-FE has been implemented into the FE code ZéBuLoN, tested for different types of external loads (tension, shear, etc.), and validated thanks to reference data from a fine conventional mesh.

REFERENCES

[1] B. Langrand, E. Deletombe, E. Markiewicz, P. Drazétic. Riveted joint modelling for numerical analysis of airframe crashworthiness. *Finite Elem. Anal. Des.*, Vol. **38**, 21–44, 2001.

[2] A. Combescure, F. Delcroix, L. Caplain, S. Espanol, P. Eliot. A finite element to simulate the failure of weld points on impact. *Int. J. Impact Eng.*, Vol. **28**, 783–802, 2003.

[3] B. Langrand, A.S. Bayart, Y. Chauveau, E. Deletombe. Assessment of multi-physics FE methods for bird impact modelling - Application to a metallic riveted airframe. *Int. J. Crashworthiness*, Vol. **7**, 415–428, 2002.

[4] N. Leconte, B. Langrand, E. Markiewicz. On some features of a plate hybrid-Trefftz displacement element containing a hole. *Finite Elem. Anal. Des.*, Vol. **46**, 819–828, 2010.

[5] F. Furgiuele, F. Cesari, D. Martini. Analisi di strutture piane in materiale eterogeneo con l'elemento finito ibrido. *XXX AIAS*, Alghero (Italy), 1093–1101, 2001.

[6] N.I. Muskhelishvili. *Some Basic Problems of the Mathematical Theory of Elasticity.* Noordhoff, Groningen, Holland, 1953.

[7] G.C. Sih. Stress distribution around an ovaloid hole under arbitrary concentrated forces. *Appl. Sci. Res.*, Vol. **12**, Sec. A, 378–390, 1964.