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SCALE-TRUNCATION MODELS FOR LARGE-EDDY SIMULATION

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Large-eddy simulation (LES) seeks to predict the dynamics of spatially filtered turbulent flows [1]. Therefore, a spatial filter is applied to the Navier-Stokes equations. In this paper we consider a box or top-hat filter. The filtering operator $u \mapsto \overline{u}$ is then given by

$$\overline{u} = \frac{1}{|\Omega_\delta|} \int_{\Omega_\delta} u(x,t)\, dx,$$

where Ω_δ denotes a part of the flow domain with diameter δ. Since this filter does not commute with the convective nonlinearity in the Navier-Stokes equations, we obtain a term that represents the effects of the residual scales of motion on the filtered flow. When this term is modeled in terms of the filtered flow, we get

$$\partial_t v + (v \cdot \nabla) v - 2\nu \nabla \cdot S(v) + \nabla p = -\nabla \cdot \tau(v), \tag{1}$$

where τ denotes the closure model and S is the symmetric part of the gradient operator. Here, the variable name is changed from $\overline{u}$ to v to stress that the closure model τ is not exact. This is crucial because information is lost in the filtering process: v possesses fewer scales of motion than the Navier-Stokes solution u. The very essence of LES is that the LES solution v is composed of eddies of size $\geq \delta$, where δ is a user-chosen length of the filter. This property enables us to solve Eq. (1) numerically when it is not feasible to compute the full Navier-Stokes solution u. Therefore, we view the closure model τ as a function of v that is to be designed to eliminate all scales of length $< \delta$.

In this paper, we address the question “when does the LES model stop the production of smaller scales of motion from continuing at the filter scale?”. This leads to the following condition

$$\int_{\Omega_\delta} \tau(v) : S(\nabla^2 v)\, dx \;=\; 4 \int_{\Omega_\delta} r(v)\, dx, \tag{2}$$

where r is the third invariant of $S(v)$, i.e., $r(v) = -\frac{1}{3}\mathrm{tr}(S^3(v)) = -\det(S(v))$. It may be emphasized that it is assumed that Ω_δ is a periodic box, so that boundary terms resulting from integration by parts vanish; see also Refs. [2]-[3] for details.

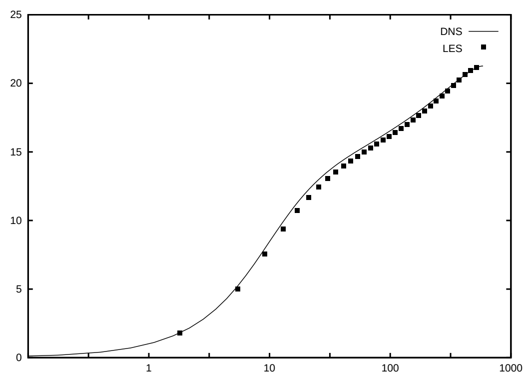
If the closure model is taken to be

$$\tau(v) - \tfrac{1}{3}\mathrm{tr}(\tau(v))\mathrm{I} \;=\; 2\nu_t S(\nabla^2 v), \tag{3}$$

the scale-truncation condition (2) becomes

$$\nu_t \;=\; \frac{\int_{\Omega_\delta} r(v)\,dx}{\int_{\Omega_\delta} q(\nabla^2 v)\,dx}, \tag{4}$$

where $q(v) = \frac{1}{2}\mathrm{tr}(S^2(v))$ is the second invariant of the strain rate tensor S. The resulting model truncates the dynamics at the scale δ set by the filter. The performance of the novel closure model has to be investigated for many cases. As a first step it was tested for turbulent channel flow at $Re_\tau \approx 590$. Fig. 1 shows a comparison of LES with DNS, where the LES-grid contains 64^3 grid points.



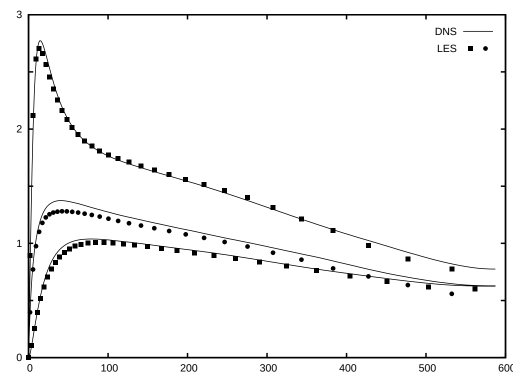


Figure 1: Compasison of LES with DNS: mean (left) and rms (right) velocities at $Re_\tau \approx 590$.

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