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SYMMETRY PRESERVATION AND VOLUME CONSISTENCY IN R-Z STAGGERED SCHEMES

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This talk is focused on the issue of symmetry preservation, energy and volume conservation and other important properties of Lagrangian hydrodynamic schemes in cylindrical geometry. Existing approaches to construct schemes in r-z will be reviewed, and the way they deal with basic physical requirements studied. Ideas will be given on how to overcome their drawbacks while leveraging their strengths, and examples of practical implementations will be shown.

The so-called area-weighted (AW) approach is popular because it easily converts any Cartesian scheme into an axi-symmetric one and because it preserves spherical symmetry on equiangular polar meshes. However, it suffers from several drawbacks. For example, using Cartesian artificial viscosity forces in the AW manner may destroy their dissipativity [1]. Even for adiabatic flows, where artificial viscosity is not employed, AW pressure forces violate the Geometric Conservation Law (GCL, quasi entropy conservation). Some of the flaws of AW can be overcome by using methods that are genuinely r-z. For example, in [2] the authors suggest zonal pressure forces that satisfy the GCL by construction, however this approach can become highly non-symmetric even on symmetric meshes, especially near the z-axis. In [1] an artificial viscosity was presented that preserves symmetry and is strictly dissipative.

With quadrilateral cells it is known that, in r-z, spherical symmetry preservation, perfect satisfaction of GCL, and total energy conservation are incompatible [3]. Being aware of this, we propose a staggered approach that tries to do its best by diminishing the GCL error to the order of entropy error. To conserve total energy by construction, we formulate the scheme using intercell fluxes. We start with the volume consistent forces from [2] and correct them so that spherical symmetry is preserved. A similar idea was developed before in the cell-centered framework, when the first order version of Maire's GCL compatible scheme [4] was symmetrized by Cheng and Shu [5].

Several ways to practically implement our symmetrization terms (for pressure as well as viscous forces) will be presented. While they are equivalent for symmetric problems on equiangular polar meshes, their effects on other mesh topologies differ, especially regarding robustness, stability and the additional assumptions required.

We also present and discuss some other approaches, differing in which of the above aspects are their primary focus. An overview of their properties and comparisons on selected numerical tests will be given.

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