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POWER SERIES ANALYSIS TO IMPROVE THE ANM CONTINUATION NEAR SIMPLE BIFURCATIONS

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First order predictor-corrector algorithms with pseudo-arclength parametrization and adaptative steplength are today the conventional method for the continuation (or path following) of solution branches to nonlinear problem depending on a parameter.

The so-called Asymptotic Numerical Method (ANM) [2] is a non-conventional continuation method which relies on the computation of high order truncated Taylor series of the solution branches

$$\mathbf{U}(a) = \mathbf{U}_0 + a\mathbf{U}_1 + a^2\mathbf{U}_2 + ... + a^N\mathbf{U}_N. \qquad (1)$$

where $\mathbf{U}_0$ is a known starting point on the branch, $\mathbf{U}_1$ the tangent vector at $\mathbf{U}_0$, $\mathbf{U}_k$ higher order derivative vectors and a the pseudo-arclength parameter. As opposed to predictor-corrector algorithms, ANM provides a local continuous representation (1) of the branch at each step of the continuation process. These series contain numerous relevant informations on the computed branch, and as claimed by Van Dyke in the mid seventies for scalar series [4], it is worth to extract this information using power series analysis in order to improve the range of utility of the representation (1) and to understand the nature and the position of the singularities of the computed solution branches.

In the present work, we analyse the power series that arise at every step of the ANM continuation algorithm and we show that when the start point $\mathbf{U}_0$ is in the neighborhood of simple bifurcation point a geometric power series always emerges in (1). Precisely, for p high enough, one has

$$\mathbf{U}_{p+1} \simeq \alpha\mathbf{U}_p \qquad \mathbf{U}_{p+2} \simeq \alpha\mathbf{U}_{p+1} \qquad ... \qquad (2)$$

This geometrical series is characterized by a common ratio α and a scale vector $\mathbf{U}_e$. By using a simplified first order perturbation approach, we evidence that the geometric series is due to a simple pole singularity indicating a nearby bifurcation. The common ratio gives the distance from the starting point to the bifurcation point and the scale vector gives the direction of the bifurcated branch at the bifurcation. Finally, we show how to use this information to accurately locate any simple bifurcation and cheaply compute the two tangents at the branch point (usefull for branch switching). All these results, presented in details in [3], are in line with previous bifurcation indicators [1] designed for ANM.

Practically, when a geometric series is detected during the continuation, it is extracted from (1) and recasted with a simple pole rational fraction. This improved representation of the solution branches not only permits to locate the bifurcation but also, the "cleaned" series recovers an optimal step length in the vicinity of the bifurcation points. Hence, the drawback that ANM continuation makes an accumulation of small size steps near bifurcation is removed.

By using several example taken from solid and fluid mechanics, we show how a systematic use of power series analysis permits to significantly improve the ANM continuation near bifurcations.

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