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ENHANCED LOW ORDER SOLID FINITE ELEMENTS USING INCOMPATIBLE INERTIA IN EXPLICIT TIME INTEGRATION FOR INSTABILITY REDUCTION

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Using the explicit central difference scheme has proven to be perfectly suitable for time integration of finite element analysis of predominantly transient problems. Many other rather slow applications including large deformations and contact are also handled very succesfully. For the usage of diagonalized mass matrices equation solving can be omitted and only vector operations need to be carried out on the global level. Hence the cost of an explicit simulation is directly related to the size of the stable time step which is bound by the maximum element eigenfrequency. Therefore operations on element level dictate the efficiency of the simulation by means of the overall computational effort.
As finite elements using low order interpolations of geometry and displacements are attractive due to their robustness and efficiency a large number of approaches to reduce or eliminate the artificial stiffness effects associated to these elements is known. One of the most prominent approaches is the method of enhanced assumed strains (EAS) suggested by Simo and Rifai [1]. The basic idea is to suppress locking effects by enhancing the strain field with additional parameters which are eliminated from the global equation system by a condensation procedure on element level. According to Bischoff and Romero [2] a generalization of this method may be applied by utilizing incompatible displacement modes that reproduce the enhanced strain field. By considering time derivatives of these incompatible displacements – incompatible velocities and accelerations – the utilization of inertia related to these modes is possible, as proposed by Mattern et al. [3]. Hence the incompatible parameters may be integrated in time with an explicit method on element level instead of being condensed out with a static elimination procedure. This is where the efficiency on element level may be affected considerably because the condensation procedure with EAS involves the solution of a nonlinear equation system when the material behaviour is getting nonlinear. However, with the introduction of incompatible displacements additional eigenfrequencies arise which are usually related to rather large stiffnesses and as a consequence reduce the size of the stable time step. This motivates scaling the

so-called incompatible mass terms in order to keep the size of the time step without considering incompatible terms. It can be shown that if the incompatible mass tends to either zero or infinity the EAS formulation resp. the original single field displacement element are retrieved. A combination of the proposed approach and the condensation procedure may be optimal to reduce the numerical effort.

Application of the suggested approach inside an explicit time integration algorithm allows treating the well known stability problem of EAS elements [4, 5] from a different point of view. The instabilities may be detected and reduced or removed analogously to the so-called hourglassing kinematics removal which is well known from explicit analysis with one point integrated element formulations. The detection is performed separately from the global equation solving in form of e.g. monitoring the kinetic energy related to any individual incompatible term inside of an element. Parameters tending to cause instabilities can be diminished or eliminated from the affected elements by simply scaling the mass related to them. By this the incompatible parameters can be partially or fully removed if any element kinematics related to them are observed. The proposed scheme is applied to solid and solid-shell finite elements known in the literature using EAS in combination with e.g. the method of assumed natural strains (ANS) or underintegrated element formulations. The EAS parameters are either treated with the proposed concept of incompatible inertia or the condensation procedure. The combination of both methods is optimized regarding numerical efficiency and numerical stability by adding incompatible inertia to terms which are known to introduce hourglassing. Benchmark examples showing instabilities are analysed under quasi-static and transient conditions in order to show the performance of the proposed scheme within explicit time integration. An important aspect is the usage of the automatic code generation tool AceGen [6] allowing an efficient and error free implementation of the proposed multitude of element formulations including variations of the treatment of the EAS parameters.

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