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# MULTIPLE MODES OF BUBBLE PROPAGATION IN PARTIALLY OCCLUDED TUBES

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When air displaces viscous fluid from simple confined geometries, e. g. cylindrical or rectangular tubes, a single propagation mode results. In contrast, recent experiments in more complex geometries (axially-uniform rectangular channels containing a centred step-like occlusion, see Figure 1) have revealed coexisting families of both steady and oscillatory propagating modes for an imposed steady driving flow rate. [1], see Figure 2.

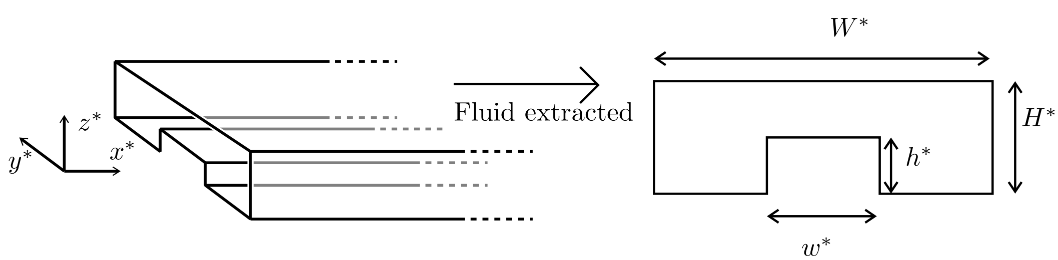


Figure 1: Schematic diagram of the channel geometry used in the experiments of [1]. The channel cross-section, shown on the right, is independent of the axial coordinate $x^*$. The dimensional outer width and height of the cross-section are $W^*$ and $H^*$ respectively. The cross-section is occluded by an obstacle of dimensional width $w^*$ and height $h^*$ placed on one of the long sides of the cross-section.

In order to understand the origin of these multiple solutions, we develop a depth-averaged (two-dimensional) model of the system by extending the work of McLean & Saffman [2] to include a smoothed version of the occlusion. The resulting equations are solved numerically using the open-source finite element library `oomph-lib` (www.oomph-lib.org). Once the problem is formulated, the library allows us to use a combination of continuation, bifurcation tracking, linear stability and time simulation to investigate the behaviour of the system.

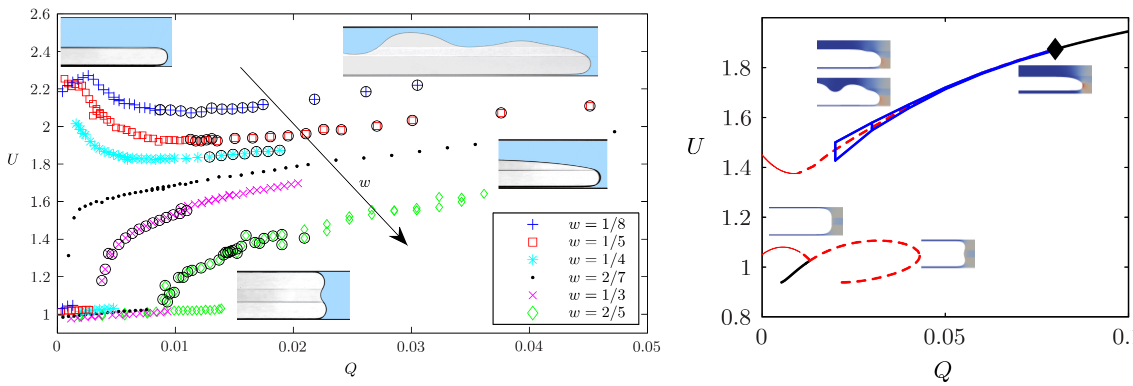


Figure 2: (Left) Experimental bifurcation diagram of possible propagation modes replotted from [1]. $Q$ is a dimensionless flow rate and $U$ is the dimensionless propagation speed of the air finger. Oscillatory propagation modes are indicated with a black circle. The step occupies half the channel height and its fractional width is given by $w = w^*/W^*$. (Right) Numerical bifurcation diagram of possible propagation modes for an obstacle occupying 15% of the channel height and 25% of its width. Unstable solutions are shown as dashed lines and the blue envelope represents the amplitude of stable oscillations.

The resulting Poisson-like equation for the fluid pressure is discretised with piecewise quadratic triangular finite elements. We use a moving-mesh method, with the mesh motion governed by treating it as an elastic solid. In order to prevent mesh deterioration affecting the interface motion, we perform a complete mesh regeneration every few steps, generating a new triangulation based on the previous discretisation of the boundary when required by an estimate of the error within the fluid domain, deformation of the elements themselves, or due to a poor discretisation of the interface.

We find that the reduced model can reproduce qualitatively all the finger propagation modes observed experimentally, see Figure 2. We analyse the bifurcation structure of the model and find that it is remarkably similar to a structure conjectured from the experimental measurements. The spatially variable channel depth modifies McLean & Saffman's model by introducing: (i) a variable mobility coefficient within the fluid domain due to variations in viscous resistance of the channel; and (ii) a variable transverse curvature term in the dynamic boundary condition that modifies the pressure jump over the air-liquid interface. We use our model to examine the roles of these two distinct effects and find that both contribute to the steady bifurcation structure, but that the transverse curvature term is responsible for the distinctive oscillatory propagation modes.

## REFERENCES


[1] A. L. Hazel, M. Pailha, S. J. Cox, & A. Juel. Multiple states of finger propagation in partially occluded channels. *Phys. Fluids* **25**, 062106, 2013

[2] J. W. McLean & P. G. Saffman. The effect of surface tension on the shape of fingers in a Hele Shaw cell. *J. Fluid Mech.* **102**, 455–469, 1981.