

ANALYSIS OF WOODEN FRAMED STRUCTURES WITH SEMI-RIGID CONNECTIONS

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Key Words: *Wood framed structures, semi-rigid connections, numerical analysis.*

In wooden structures, analysis of structural systems has followed the progressive understanding about the connections behavior. However, classical structural methods do not consider the deformation of the connections and its effects are not widely known. The analysis of framed structures with semi-rigid connections has two aspects: the structural model and the connection behavior model. In this work, we present a parametric analysis of wooden framed structures with semi-rigid connections with general nonlinear behavior, in which the connection properties are varied and their effects on different structures are observed.

The contribution of the deformation of connections in the behaviour of structures has been object of concern since experimental results have showed its importance. The theories about semi-rigid connections followed the development of matrix analysis of framed structures. With the advances in the finite element analysis commercial programs, the most natural method to account for the contribution of connection deformation is by means of connections elements. The method for the computation of the connection deformation is one of the aspects of the analysis of framed structures with semi-rigid connections. The aspect explicitly related to the material is the description of the behaviour of the connections elements, accounting for the relationship between the nodal displacements and forces. For wood, previous works have been made by Jensen and Larsen [1], for example. The process of design may lead to different connections with different effects in the structure, in both quantitative and qualitative senses, depending respectively in the stiffness and the geometry of the structure. We may expect that the optimum design of structures be in a large extent related to the design of the connection. In this work, we make use of a parametric analysis varying the moment resisting connection behaviour to analyse the effects of the connection properties on the structure response.

The first order analysis of framed structures has its foundations on the Principle of Virtual Work (PVW). Nonlinear analysis including deformations of the connections is based on the same theory, excepted that the contribution of the connections is included in the domain, by means of connections elements. Internal virtual work due to the bars deformations are computed by means of the simplifications of the classical theory of bending, and the application of the PVW to a generic bar element. leads to:

$$\mathbf{K}_b^e \mathbf{u}_b^e = \mathbf{a}_b^e \quad (1)$$

where \mathbf{K}_b^e is the stiffness of the bar element, and \mathbf{u}_b^e and \mathbf{a}_b^e are respectively the nodal displacements and the nodal forces in the bar ends [2]. Internal virtual work due to connections deformations are computed by means of simplifications in the description of the internal stresses and strains, in such a way that the application of the PVW to a generic connection element leads to:

$$\mathbf{f}_c^e(\mathbf{u}_c^e) = \mathbf{a}_c^e \quad (2)$$

where \mathbf{f}_c^e is a vector with nonlinear functions of \mathbf{u}_c^e , and \mathbf{u}_c^e and \mathbf{a}_c^e are respectively the nodal displacements and the nodal forces in the connections ends. Equations (1) and (2) may be assembled into a global equation equivalent to the equation of PVW. Prior linearization of equation (2) may be accomplished by means of the Newton-Raphson method, and a recursive equation is obtained:

$$\mathbf{K}_c^e(\mathbf{u}_c^{e(k)}) \Delta \mathbf{u}_c^{e(k+1)} = \mathbf{a} - \mathbf{f}_c(\mathbf{u}_c^{e(k)}) \quad (3)$$

where k is the number of the iteration and:

$$\mathbf{K}_{c(ij)}^e = \frac{\partial \mathbf{f}_{c(i)}^e}{\partial \mathbf{u}_{c(j)}^e} \quad (4)$$

The relationship between nodal loads and displacements for the connections elements may be developed more commonly by means of an experimental procedure or an approximate theoretical method. According to this last method, the nodal forces are assumed to be statically equivalent to the set of forces acting in each connector and the nodal displacements cinematically equivalent to slip of each connector. The virtual work of the forces on the virtual relative displacements of the connectors is theoretically assumed to be equivalent to the virtual work of the stresses on the virtual strains in the connection volume. The force in one connector is related to the relative displacements in the connector by means of a nonlinear relationship of the type, originally proposed by Foschi[3].

With these bases, the relationship given by equation (2) may be developed. The construction of the functional form of the connections behaviour according to these principles was made by Jensen and Larsen[1] and also by Santana and Mascia[4].

As an application of this method consisted in designing a framed structure with minimum and maximum typical dimensions for commercial use, with L=8.0 and 16.0 m. Initially, the structures were designed to characteristic forces determined according to the classical theory of framed structures. The prior results obtained in such a way were used for the design of the connections. The initial stiffness of the connections and the parameters that define its behaviour were determined. Then, the analysis of each structure was repeated according to the model described. The parametric analysis involved the assignment of different connections for the same demand of strength, and the consideration of non-linear and approximate linear behaviour for the connections as well.

From these results, we observe that the linear analysis leads to results very close to those obtained with non-linear analysis, with major deviations for the displacements. The results are dependent on the connector parameters and the resultant global initial stiffness of the connection. By comparing the rigid and the semi-rigid results, we see that the displacements are significantly affected, as well as the bending moment distribution. On the other hand, forces are not affected in much extent.

References

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