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BLOCKWISE ADAPTIVITY FOR DIFFUSE-INTERFACE TUMOR-GROWTH MODEL

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In this contribution, we consider a thermodynamically consistent four-species model of tumor growth [2]. Employing the continuum theory of mixtures, the tumor-growth model has been derived with diffuse interfaces. The chosen form of the Helmholtz free-energy leads to equations of the reactive Cahn-Hilliard type. The formulation involves a system of nonlinear fourth-order partial-differential equations, for which the energy of the system is non-increasing and total mass is conserved. A gradient-stable time-stepping scheme with second-order accuracy is developed to inherit these two characteristics of the system at the discrete level [4]. Mixed finite elements are used for spatial discretization.

Solutions of the diffuse-interface tumor-growth model exhibit alternating fast and slow variations in time and impose severe demands on the resolution of the interface. It is therefore of prime importance to apply adaptivity both in space and in time to efficiently simulate the entire dynamics of the system. A blockwise adaptive approach using adjoint solutions for time-dependent problems has been discussed and developed by Carey et al [1]. Following this idea, we propose a novel duality-based two-level error estimation framework [3]. Several numerical experiments for the tumor-growth model are presented, to demonstrate the proposed framework.

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