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NUMERICAL AND ANALYTICAL HOMOGENEIZATION OF THE PERMEABILITY OF POROUS MEDIA

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The permeability in Darcy's law upscales from viscous Stokes flow through porous media. Bounds and estimates on the permeability based on a limited number of morphological and statistical parameters to describe the microstructure are valuable tools to understand the complex relation between pore microstructure morphology. One major advantage of bounds and estimates is to provide closed-form or semi-analytical expressions which can be readily used by engineers.

Bounds are typically based on morphological and statistical representations of the pore and solid phases, such as particle shape and inter-penetration and multi-point volume and surface correlation functions [1, 2, 3]. Sharp bounds can be used as estimates.

Other estimates derive from in-cell models, based on the resolution of a boundary value problem for viscous flow in a simplified morphological pattern [4, 5]. Viscous flow within concentric spheres has been solved exactly, whereas flow within confocal spheroids has only been estimated for high porosity and moderate aspect ratios.

However, most statistical models face technical difficulties to explicit particle anisotropy effects. Additionally, the optimal choice of boundary conditions for in-cell models is also to be determined. Finally, the accuracy of estimates and bounds is difficult to check in practice.

First, a focus on particle anisotropy is proposed. To this end, new semi-analytical in-cell estimates are proposed for a wide range of porosity and particle aspect ratio by providing accurate bounds on the viscous flow within confocal spheroids. Two types of boundary conditions are tested.

Second, an efficient, mesh-free, FFT-based full field method [6] is used to test the permeability of microstructures previously out of reach numerically. Tested microstructure include randomly distributed penetrable spherical or oblate/prolate solid grains. The ease of use and efficiency of our FFT-based method allows to consider a great number of random microstructure realizations. FFT methods where originally proposed in linear elasticity [7] and adapted to viscous flow [8]. Our numerical schemes benefits from the transposition to viscous flow of recent improvements of FFT-based methods in linear elasticity [9] by the use of a Hashin-Shtrikman variational framework.

REFERENCES

[1] S. Prager. Viscous flow through porous media. *Phys. Fluids*, Vol. **4**, N. 12, 1477–1482, 1961.

[2] M. Doi. A New Variational Approach to the Diffusion and the Flow Problem in Porous Media. *J. Phys. Soc. Jpn.*, Vol. **40**, 567–572, 1976.

[3] J. Rubinstein, S. Torquato. Flow in random porous media: mathematical formulation, variational principles, and rigorous bounds. *J. Fluid Mech.*, Vol. **206**, 25–46, 1989.

[4] G. Dassios, M. Hadjinicolaou, F.A. Coutelieris, A.C. Payatakes. Stokes flow in spheroidal particle-in-cell models with Happel and Kuwabara boundary conditions. *Int. J. Engng Sci.*, Vol. **33**, N. 10, 1465–1490, 1995.

[5] C. Boutin. Study of permeability by periodic and self-consistent homogenisation. *Eur. J. Mech. A/Solids*, Vol. **19**, 603–632, 2000.

[6] F. Bignonnet and L. Dormieux. FFT-based bounds on the permeability of complex microstructures. Submitted to *Int. J. Num. Anal. Meth. Geomech.*

[7] H. Moulinec, P. Suquet. A fast numerical method for computing the linear and non linear properties of composites. *C.R. Académie Sciences*, Vol. **2**, N. 318, 1417–1423, 1994.

[8] V. Monchiet, G. Bonnet, G. Lauriat. A FFT-based method to compute the permeability induced by a Stokes slip flow through a porous medium. *C.R. Mécanique*, Vol. **337**, 192–197, 2009.

[9] S. Brisard, L. Dormieux. FFT-based methods for the mechanics of composites: A general variational framework. *Comp. Mat. Sc.*, Vol. **49**, 663–671, 2010.