11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

DISCONTINUOUS GALERKIN METHODS FOR SOLVING HELMOLTZ ELASTIC WAVE EQUATIONS FOR SEISMIC IMAGING

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Key words: *Discontinuous Galerkin methods, Hybridization, Elastic waves propagation.*

One of the most used seismic imaging methods is the full wave inversion (FWI) method which is an iterative procedure whose algorithm is the following. Starting from an initial velocity model, **a)** one computes the solution of the wave equation for the N sources of the seismic acquisition campaign and, **b)** one evaluates, for each source, a residual defined as the difference between the wavefields recorded at receivers on the top of the subsurface during the acquisition campaign and the numerical wavefields. Then, **c)** one computes the solution of the wave equation using the residuals as sources, and **d)** one updates the velocity model by cross correlation of images produced at steps **a)** and **c)**. Finally, the different steps **a)** to **d)** are repeated until convergence of the velocity model is achieved. We then have to solve $2N$ wave equations at each iteration. The number of sources, N, is usually large (about 1000) and the efficiency of the inverse solver is thus directly related to the efficiency of the numerical method used to solve the wave equation.

Seismic imaging can be performed in the time domain or in the frequency domain regime. We focus here on the second setting. The drawback of time domain is that it requires to store the solution at each time step of the forward simulation. The difficulties related to frequency domain inversion lie in the solution of huge linear systems, which cannot be achieved today when considering realistic 3D elastic media, even with the progress of high-performance computing. In this context, the goal is to develop new forward solvers that reduce the number of degrees of freedom without hampering the accuracy of the numerical solution.

We consider here discontinuous Galerkin (DG) methods which are more convenient than finite difference methods to handle the topography of the subsurface. Moreover, they are more adapted than continuous Galerkin (CG) methods to deal with $hp-$adaptivity. This last characteristics is crucial to adapt the mesh to the different regions of the subsurface which is generally highly heterogeneous. Nevertheless, the main drawback of classical DG methods is that they are expensive because they require a large number of degrees of freedom as compared to CG methods on a given mesh.

In this work we consider a new class of DG method, the hybridizable DG (HDG) method (see [1] for a more details). Instead of solving a linear system involving the degrees of freedom of all volumic cells of the mesh, the principle of HDG consists in introducing a Lagrange multiplier representing the trace of the numerical solution on each face of the mesh. Hence, it reduces the number of unknowns of the global linear systems and the volumic solution is recovered thanks to a local computation on each element. HDG methods have been considered in some recent works, for example, for the solution of the elastodynamic equations in the time domain [2] and for Maxwell's equations [3].

We compare the performances of the HDG method with those of classical nodal DG methods, like methods used in [4] and [5], and we present our first results using a HDG method for the first-order form of the elastic wave propagation equations for 2D realistic test-cases.

The authors acknowledge the support by the Inria-Total strategic action DIP (dip.inria.fr).

REFERENCES

[1] R.M. Kirby, S.J. Sherwin, B. Cockburn. To CG or to HDG: A Comparative Study. *Journal of Scientific Computing* , Vol. **51**, 183–212, 2012.

[2] N.C. Nguyen, J. Peraire, B. Cockburn. High-order implicit hybridizable discontinuous Galerkin methods for acoustics and elastodynamics. *Journal of Computational Physics*, Vol. **230**, 3695–3718, 2011.

[3] S. Lanteri, L. Li, R. Perrussel. Numerical investigation of a high order hybridizable discontinuous Galerkin method for 2d time-harmonic Maxwell's equations. *The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. **32**, 1112–1138 , 2013.

[4] M. Dumbser, M. Käser. An arbitrary high-order discontinuous Galerkin method for elastic waves on unstructured meshes - II; The three-dimensional isotropic case. *Geophysical Journal International*, Vol. **167**, 319–336, 2006.

[5] M. El Bouajaji, S. Lanteri. High order discontinuous Galerkin method for the solution of 2D time-harmonic Maxwells equations. *Applied Mathematics and Computation*, Vol. **219**, 7241–7251, 2013.