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AN APPROXIMATION FRAMEWORK DEDICATED TO PGD-BASED NONLINEAR SOLVER

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When dealing with high-fidelity models, the number of degrees of freedom can lead to systems so large that direct techniques are unsuitable. Reduced order modeling (ROM) seeks to reduce the computational complexity and computational time of large-scale systems by approximations of much lower dimension. Several techniques have been developed in order to provide reduced-order models of systems such as POD-based method (see e.g. [1]) or PGD technique [2].
Nevertheless, one of the major issues of ROM is to deal efficiently with non-linearities. Indeed, some linearization techniques must be used and the computation of the reduced operators can represent the most important cost of the resolution. It is, for instance, the case when dealing with a Galerkin projection to obtain the ROM once a reduced order basis (ROB) of the problem is built. In order to solve this issue, some interpolation methods have been introduced in the context of the Proper Orthogonal Decomposition and Reduced Basis in order to reduce the dimensions of these operations when dealing with non-linear and/or non-affine parametrized PDEs [4, 5].
The aim of this work is to take advantage of a dimension reduction method, called Reference Points Method (RPM, introduced in [2]), in order to decrease the computational complexity of algebraic operations in the framework of another model reduction method: the Proper Generalized Decomposition (PGD). Roughly, the PGD technique consists in seeking the solution of a problem in a relevant ROB which is generated automatically and on-the-fly by the LATIN method [3]. The LATIN method is an iterative strategy which generates the approximations of the solution over the entire time-space-parameter domain by successive enrichments. At a particular iteration, the ROB which has been already formed is first used to compute a projected ROM and find a new approximation of the solution. If the quality of this approximation is not sufficient, the ROB is enriched by determining a new functional product using a greedy algorithm.
The present work focuses on a further reduction of the computational cost and storage, thanks to the introduction of a new approximation framework dedicated to PGD-based nonlinear solver. It is based on the concept of *reference* times, points and parameters

and is used to define a compressed version of the data. Compared to similar techniques [4, 5] this is not an interpolation technique but an algebraic framework allowing to give an inexpensive first approximation of all quantities in a separated variable form by explicit formulas. **Fig.1** illustrates the reconstruction f' of a function f obtained by multiplying two random functions defined over the time-space domain $\Omega \times I$ with an error ξ:

$$\xi = \frac{\|f' - f\|_{I\times\Omega}}{\|f\|_{I\times\Omega}} \; < \; 1\%, \qquad\qquad \|f\|^2_{I\times\Omega} = \int_{I\times\Omega} f^2 \, d\Omega dt. \tag{1}$$

The new framework has been introduced in the PGD-based nonlinear solver to compute

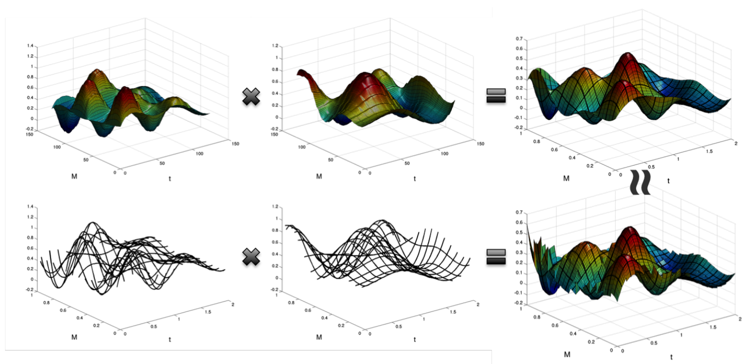

Figure 1: The exact f (up) and its approximation f' rebuilt by 13 ref. instants and 13 ref. points (down).

some repetitive algebraic operations. Some examples of the computational efficiency of this new approach will be given to show that the gain in the complexity for this kind of operation, that is computed frequently along the iterations, is in the order of the ratio $\frac{\Delta t_{ref}}{\Delta t}$, where Δt (resp. Δt_{ref}) is the initial (resp. reference) time step.

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