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LEVEL SET TOPOLOGY OPTIMIZATION BASED ON SEQUENTIAL LINEAR PROGRAMMING

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1. INTRODUCTION

Level set methods have become increasing popular for solving topology optimization problems, as they produce solutions with clear boundaries [1-3]. The conventional level set optimization method uses shape sensitivity analysis to define a velocity function that moves the boundary to a more optimal position with respect to the objective and constraints. Constraints can be handled though the Lagrange multiplier method [2,3], although including multiple constraints can be difficult. Furthermore, the conventional level set optimization method is based on shape optimization and it is not easily extended to include other design variables beyond boundary position. Many practical engineering optimization problems involve multiple constraints and have many design variables, including the boundary position and shape. A new method is developed that utilizes boundary integrals and linear programming to introduce multiple constraints and additional design variables into the conventional level set based optimization method.

2. THE SEQUENTIAL LINEAR PROGRAMMING LEVEL SET METHOD

To illustrate the new method, we use a generic optimization problem where one of the design variables is the position of the structural boundary:

$$\text{Minimise: } f(\Omega, x), \qquad \text{Subject to: } g_i(\Omega, x) = \overline{g}_i \qquad i = 1, 2, \cdots, m \tag{1}$$

where f is the objective function, g_i are the constraint functions, m is the number of constraints, Ω is the domain of the structure, which can be characterised by the position of its boundary Γ, and x is a vector of additional design variables. Inequality constraints can be included in the above problem by using slack variables.

It is assumed that the first derivative of the objective and constraints with respect to the additional design variables can be computed. The remaining variable in the problem is the shape of the structure, characterized by the position of its boundary. Shape sensitivity analysis provides information about how a function changes with respect to a movement of the boundary and takes the form of a shape derivative. Over a time step, Δt, the change in a function is:

$$\Delta t \frac{\partial f(\Omega, x)}{\partial \Omega} = \Delta t \int_{\Gamma} \left(s_f \cdot V \right) d\Gamma = \int_{\Gamma} \left(s_f \cdot z \right) d\Gamma \tag{2}$$

where s_f is a shape sensitivity and V is a velocity function acting normal to the boundary. The time step can be eliminated by defining a boundary movement function: $z = \Delta t \cdot V$. The shape sensitivity and boundary move functions vary along the boundary and are usually assumed smooth. Therefore, the shape derivative is characterised as a boundary integral involving a boundary move function. We assume that the shape sensitivity function can be computed and are known. However, we are free to choose the boundary move function.

To evaluate the integral in (2), the boundary is discretized into a number of segments. If the shape sensitivity and boundary move functions are assumed piece-wise constant, then the discretization of the shape derivative for the objective function can be written as:

$$\Delta t \frac{\partial f(\Omega, x)}{\partial \Omega} \approx \sum_{j=1}^{n} \left(s_{f,j} \cdot z_j \cdot l_j \right) = c^T z \quad , \quad c_j = s_{f,j} \cdot l_j \tag{3}$$

where l_j is a discrete segment length (or surface area in 3D) for segment j, $s_{f,j}$ is a discrete value of the shape sensitivity, and n is the number of discrete segments. The same approach can be used to compute the boundary integrals for each constraint, where $s_{i,j}$ is a discrete value of the shape sensitivity for constraint i and the boundary integral coefficients are: $a_{i,j} = s_{i,j} \cdot l_j$.

For topology optimization only (i.e. no additional design variables), an "optimal" choice for the boundary movement function is one that maximises the reduction in the objective function (or minimises the shape derivative) whilst meeting the constraints, or at least moving towards a feasible solution. This can be achieved by solving the following Linear Programming (LP) sub-problem:

$$\text{Minimise: } c^T z, \qquad \text{Subject to: } a_i^T z = \overline{g}_i - g_i, \qquad i = 1, 2, \cdots, m \qquad z_{\min} \le z \le z_{\max} \tag{4}$$

where z_{min} and z_{max} are side constraints on the boundary movement, which are usually dictated by the Courant–Friedrichs–Lewy condition for stability or by the limits of the design domain. This approach was tested and reasonable results were obtained. However, the solutions did not possess smooth boundaries as the boundary move function is not guaranteed to be smooth, even if the shape sensitivity functions were smooth. The LP sub-problem (4) has too much freedom in choosing z. Therefore, an alternative strategy is introduced where the boundary move function is defined as a linear combination of the smooth shape sensitivity functions:

$$z_j = \lambda_f s_{f,j} + \sum_{i=1}^{m} \lambda_i s_{i,j} \quad , \quad z_{j,\min} \le z_j \le z_{j,\max} \tag{5}$$

where λ are weights for each shape sensitivity function. Therefore, the sub-problem during each main iteration of the level set method is to find the values of λ that produce a boundary movement function that minimises the shape derivative of the objective and meets the constrains.

The side constraints on the boundary movement are enforced after z is computed from the specified λ values in equation (5). Therefore, the change in the objective or a constraint is not a linear function of the λ values and they cannot be directly obtained by solving a LP sub-problem. However, the function is likely to be nearly linear, as it only becomes non-linear when the side constraints on z are active. Therefore, we can use Sequential LP (SLP) to solve for λ. We term this method the SLP level set topology optimization method.

The SLP level set topology optimization method has several numerical issues to address including the obtainment of appropriate limits and first order gradients for the λ values and a method to handle non-linear constraints. Efficient and robust procedures have been developed to address each issue and will be presented.

3. EXAMPLE

The SLP level set method is demonstrated by first solving the classic minimization of compliance problem subject to a volume constraint. The final compliance value is then used as the constraint in the dual problem of volume minimization while being subject to a compliance upper limit. The results for a 2:1 ratio cantilever are shown in Fig 1. The final structures are very similar and the constraints are satisfied in both problems. Furthermore, the final volume fraction and compliance values are within 0.3%.

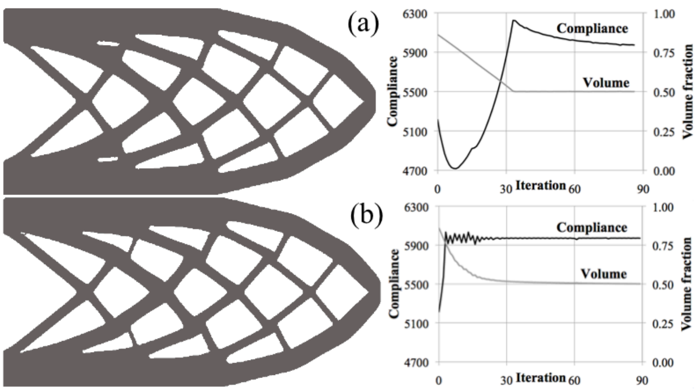

Fig. 1. Cantilever, minimization of: a) compliance b) volume.

4. CONCLUSION

The example shows that the SLP level set topology optimization method can solve a problem with a single compliance constraint. The presentation will include the details of the SLP level set topology optimization method and its application to solve problems with various constraints in 2D and 3D and on unstructured grids.

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