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EFFICIENT REDUCTION OF APPROXIMATION ERRORS BY MEANS OF MULTIPLE ENRICHED BASIS FUNCTIONS

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We propose a new generalized finite element method which efficiently reduces the error in the energy norm of the approximate solution. We do so by employing multiple enriched basis functions. A similar strategy has been proposed in [1, 2]. Such improvements were, however, often not realizable due to the ill-posedness of the constrained stiffness matrix corresponding to the basis in question. A modification is proposed which reformulates the basis and renders the constrained stiffness matrix well-posed. As a result, the numerical solution is not perturbed by round-off errors and approximates the exact solution efficiently.

To demonstrate the proposed method, we apply it to the one-dimensional screened Poisson equation

$$\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} - \alpha^2 u = 0,$$

where α is a problem dependent parameter. Solving the corresponding discrete problem, with $\{\chi_l\} := \{x^2, x^4, x^6, x^8\}$ as global enrichment functions, we can compare the condition numbers and relative errors as depicted in Figure 1.

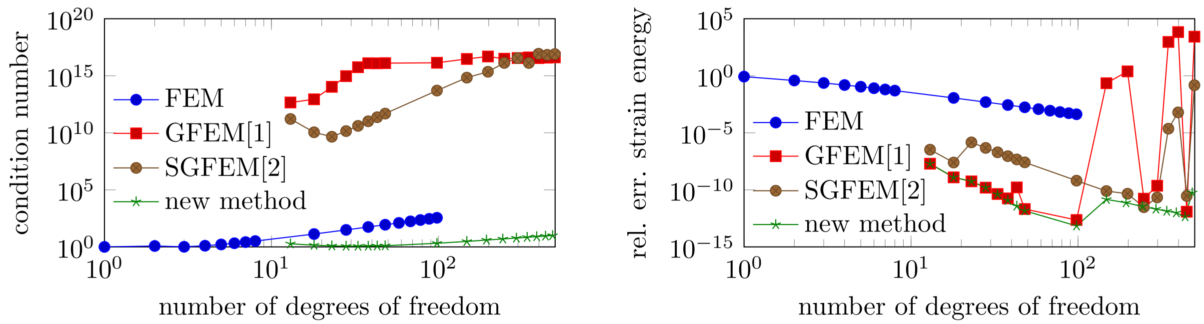

Figure 1: Comparison of condition numbers and relative errors in the strain energy for different methods (lower is better).

The standard generalized finite element method [1] and the stable generalized finite element method [2] result in large condition numbers for the constrained stiffness matrix. As a consequence, the numerical solution is perturbed by round-off errors. In contrast, the proposed method results in a low condition number and the solution is not perturbed. Similar improvements are realized when using multiple local and/or global (non)polynomial enrichment functions.

The new method is flexible and has beneficial properties. It is applicable to two- and three-dimensional problems. Multiple partitions of unity can be used. Both global and local enrichment functions can be employed without problems in the blending elements. Basis functions with a negligible contribution to the solution are automatically removed.

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