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An Explicit Dynamic Method for a Discrete Element Model using the Principle of Hybrid-type Virtual Work

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ABSTRACT

In the analysis of a dynamic fracture problem, the use of a discrete element model for the rigid body spring model (RBSM) is effective [1]. RBSM was developed as a numerical model for generalized limit analysis in plasticity, in which the structure to be analyzed is idealized as an assemblage of rigid bodies connected by normal and tangential springs.

On the other hand, by using the principle of hybrid-type virtual work, the authors have applied the explicit method to each element [2]. In this paper, we illustrate the formulization of RBSM that has been expanded to include the distinct element method (DEM) [3].

We introduce a subsidiary condition into the framework of the variational equation with Lagrange multipliers $\boldsymbol{\lambda}$, such that the hybrid-type virtual work equation can be described as follows about the M subdomain and N intersection boundary:

$$\sum_{e=1}^{M}\left(\int_{\Omega^{(e)}}\boldsymbol{\sigma}:\mathrm{grad}(\delta\boldsymbol{u})dV-\int_{\Omega^{(e)}}\boldsymbol{f}\cdot\delta\boldsymbol{u}\,dV-\int_{\Omega^{(e)}}\boldsymbol{f}_{\alpha}\cdot\delta\boldsymbol{u}\,dV\right)$$
$$-\sum_{s=1}^{N}\left(\delta\int_{\Gamma_{<s>}}\boldsymbol{\lambda}\cdot(\tilde{\boldsymbol{u}}^{(a)}-\tilde{\boldsymbol{u}}^{(b)})\,dS\right)-\int_{\Gamma_{\sigma}}\hat{\boldsymbol{t}}\cdot\delta\boldsymbol{u}\,dS=0\qquad{}^{\forall}\delta\boldsymbol{u}\in\mathbb{V}$$

This equation implies that the Lagrange multiplier $\boldsymbol{\lambda}$ is the surface force on the boundary in the subdomain. Hence, the surface force is defined as follows:

$$\boldsymbol{\lambda}_{<ab>}=\boldsymbol{k}\cdot\boldsymbol{\delta}_{<ab>}$$

Here, $\boldsymbol{\delta}_{<ab>}$ is the relative displacement at the boundary, and $\boldsymbol{k}$ is the penalty function. The equation of motion, discretized about space by substituting the above relationships, is obtained as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{U}}+\boldsymbol{K}\boldsymbol{U}=\boldsymbol{P}$$

$$\begin{bmatrix}\boldsymbol{M}_{dd} & \boldsymbol{M}_{d\varepsilon}\\ \boldsymbol{M}_{\varepsilon d} & \boldsymbol{K}_{\varepsilon\varepsilon}\end{bmatrix}\begin{Bmatrix}\ddot{\boldsymbol{d}}\\ \ddot{\boldsymbol{\varepsilon}}\end{Bmatrix}+\begin{bmatrix}\boldsymbol{K}_{dd} & \boldsymbol{K}_{d\varepsilon}\\ \boldsymbol{K}_{\varepsilon d} & \boldsymbol{K}_{\varepsilon\varepsilon}+\boldsymbol{D}\end{bmatrix}\begin{Bmatrix}\boldsymbol{d}\\ \boldsymbol{\varepsilon}\end{Bmatrix}=\begin{Bmatrix}\boldsymbol{P}_{d}\\ \boldsymbol{P}_{\varepsilon}\end{Bmatrix}$$

Here, as shown in Figure 1, we expand element (1) and the adjoining element. In this case, the integration on the boundary edge, with a focus on element (1), is only relevant to elements (2)–(4). Therefore, the other elements are not relevant to the simultaneous equations. From the above relationship, the stress element can be obtained by using the surface forces at the element boundary, which can be expressed as follows:

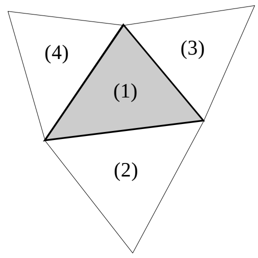

Figure 1 Element (1) and adjoining element

$$\boldsymbol{M}^{(e)}\ddot{\boldsymbol{U}}^{(e)} = \boldsymbol{P}_d^{(e)} - \oint_{\Gamma^{(e)}} \boldsymbol{N}_d^{(e)} \boldsymbol{t}^{(e)}\, d\Gamma$$

Thus, the equation of motion can be computed for each element. The element acceleration can be obtained as the resultant of the contact forces in a manner similar to the DEM approach. Moreover, we can update the position of each element by looping Δt in these calculations, as follows:

$$\ddot{\boldsymbol{U}}^{n+1} = \boldsymbol{M}^{-1}\tilde{\boldsymbol{P}}^{n}$$

$$\dot{\boldsymbol{U}}^{n+1} = \dot{\boldsymbol{U}}^{n} + \ddot{\boldsymbol{U}}^{n+1}\Delta t \qquad \boldsymbol{U}^{n+1} = \boldsymbol{U}^{n} + \dot{\boldsymbol{U}}^{n+1}\Delta t$$

The blocks in a blocky system can touch each other at their boundaries. We propose a new RBSM approach for contact types, namely, the edge-to-edge contact shown in Figure 2. In the contact state, the amount of penetration in the normal and shear directions, δn and δs, respectively, are defined as follows:

$$\delta_n = (y_m - y_c)\frac{x_{b_2} - x_{b_1}}{L_b} - (x_m - x_c)\frac{y_{b_2} - y_{b_1}}{L_b}$$

$$\delta_s = (x_m - x_c)\frac{x_{b_2} - x_{b_1}}{L_b} + (y_m - y_c)\frac{y_{b_2} - y_{b_1}}{L_b}$$

This process reduces the computational cost and simplifies the program. Moreover, in comparison with an analytical solution, our new approach has been shown as being applicable and accurate.

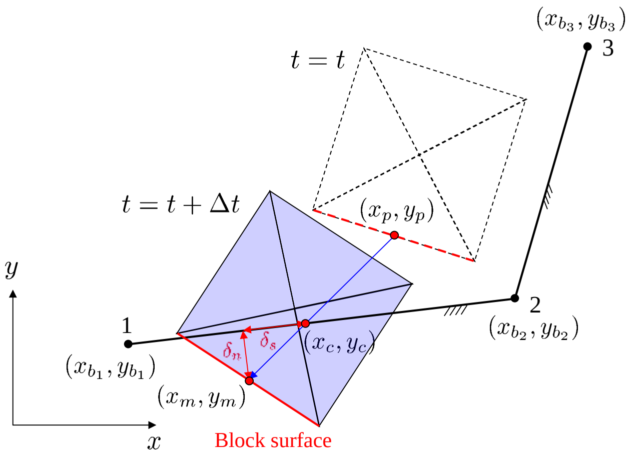

Figure 2 Contact process

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