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A COMPLETE MODEL FOR THE ANALYSIS OF THERMOELASTIC BEHAVIOUR IN MICROBEAMS

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This work deals with the numerical evaluation of thermoelastic damping (TED) in microstrucures. Being an intrinsic loss of energy, the TED mechanisms are hardly controllable. It follows that a thorough knowledge of the phenomena is required, especially for the design of devices with an high quality factor.

Within an elastic continuum, the compression and stretching of the fibres arising from a mechanical deformation result in thermal stresses. The mechanical behaviour itself is related to the temperature distribution and therefore the modelling becomes a little bit involved.

We focus on a microstrucure in which the main component is represented by an electrically-actuated microbeam that are widely used in MEMS applications.

The mechanical equation, developed under the assumptions of the nonlinear Euler-Bernoulli theory, presents two different kind of nonlinearities accounting for a second order axial stretch and for the electric forcing term. The use of the strain gradient elasticity theory permits us to take into account size-dependent phenomena in the microstructure; since the formulation is based on the modified theory of [1], the equation is enriched with a sixth-order differential term. Furthermore, we add the thermal phenomena to the formulation of the non-classical mechanical problem; the thermoelastic material is assumed to be homogeneous and isotropic, with constant material properties.

The thermo-mechanical behaviour, for the considered electrically-actuated microbeam problem, is described by a system of two coupled partial differential equations. Indicating with $w = w(x,t)$ and $\theta(x,z,t)$ the transversal deflection of the microbeam and its temperature distribution respectively, the governing system in its dimensionless form is given by:

$$
\begin{cases}
\dfrac{\partial^2 w}{\partial t^2} + c\dfrac{\partial w}{\partial t} + \dfrac{\partial^4 w}{\partial x^4} - \alpha_3 \dfrac{\partial^6 w}{\partial x^6} - \left[N + \dfrac{\alpha_1}{2} \displaystyle\int_0^1 \left(\dfrac{\partial w}{\partial x} \right)^2 dx + \beta \left(1 + t_1 \dfrac{\partial}{\partial t} \right) \displaystyle\int_0^1 \left(\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \theta dz \right) dx \right] \dfrac{\partial^2 w}{\partial x^2} + \\
\qquad + \gamma\beta \left(1 + t_1 \dfrac{\partial}{\partial t} \right) \displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(z \dfrac{\partial^2 \theta}{\partial x^2} \right) dz = \dfrac{\alpha_2 V^2}{(1-w)^2} \left(1 + 0.265\eta^{3/4} (1-w)^{3/4} + 0.53\eta\sqrt{\gamma}\sqrt{1-w} \right) \\
k \left(\mathcal{L}^2 \dfrac{\partial^2 \theta}{\partial x^2} + \dfrac{\partial^2 \theta}{\partial z^2} \right) + R + t_0 \dfrac{\partial R}{\partial t} = \mathcal{C} \left(t_0 + t_2 \right) \dfrac{\partial^2 \theta}{\partial t^2} + \mathcal{C} \dfrac{\partial \theta}{\partial t} + t_0 \beta \dfrac{\partial^4 w}{\partial t^2 \partial x^2} + \beta \dfrac{\partial^3 w}{\partial t \partial x^2}.
\end{cases}
$$

The proposed model make use of the work of [2] in which the classical thermoelasticity (CTE), the Lord-Shulman theory (LS) and the Green-Lindsday theory (GL), are unified by introducing some global parameters (see Table 1).

The microbeam is clamped at both edges, $w = w' = w'' = 0$ at $x = 0, 1$, and the clamped supports are maintained at constant temperature. Moreover we consider adiabatic boundary conditions on the beam surfaces under the assumption that the conduction of heat within the beam is faster than the exchange of heat with the environment.

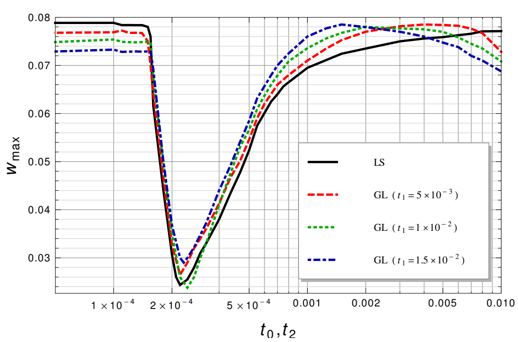

Figure 1: TED as function of the relaxation times.

case	model
$t_0 = t_1 = t_2 = 0$	classical thermoelasticity
$t_1 = t_2 = 0$	Lord-Shulman model
$t_0 = 0$	Green-Lindsay model

Table 1: Thermoelastic models summary. When $t_0 = t_1 = t_2 = 0$ the system reduces to the classical coupled thermoelasticity. If $t_0 = 0$ and $t_1 \neq 0$,$t_2 \neq 0$ we obtain the GL theory. Imposing $t_1 = t_2 = 0$ with $t_0 \neq 0$ the LS model is obtained.

Comparisons between the classical theory and the generalized model are carried out using numerical simulations. The preliminary results show how the TED mechanism is affected by the relaxation times t_i. It is shown in Figure 1 the decrement of the dimensionless deflection as function of the relaxation times: using both the LS and GL models it can be seen a larger energy dissipation phenomenon for specific values of t_i. For low values of t_i the results coincide with the classical theory. Increasing t_0, the two generalized model lead to different behaviours; the amplitude of t_1 influences the approach towards the characteristic peak. The investigation enlightens how the LS and GL models, both based on a hyperbolic energy equation, are structurally different.

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