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BAYESIAN DATA ASSIMILATION FOR NAVIER-STOKES WITH THE LEAST-SQUARES FINITE-ELEMENT METHOD

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Increasingly there is a demand for numerical methods for fluids capable of assimilating experimental data from wind-tunnel measurements into CFD simulation. This can be done with different methods towards several goals: identifying unmeasured variables such as pressure from particle-image velocimetry (PIV) velocity field measurements; increasing the time-resolution of measurement data; filling spatial gaps in PIV data [1]; and interpolating multi-variate fields with PDE constraints [2]. These methods may be seen as physics-based post-processing techniques. In each case the ability of simulation to smoothly approximate the flow-field in the full domain in all variables, or identify an unmeasured quantity of interest, increases the value of the measurement. An increase in the amount of data provided by modern experimental techniques drives a trend toward increasing demand for effective post-processing tools, with an increasing amount of physical fidelity [3].

In this paper we investigate a stochastic approach for assimilating experimental velocity measurements into Navier-Stokes flow based on a Least-Squares Finite-Element Method (LSFEM) [4,5]. Concisely: given a system of PDEs with boundary conditions, LSFEM methods define a scalar functional whose minimum is the solution of the system. The functional is defined as a sum of e.g. L_2-norms of the residuals of the original PDE and optionally BCs. These terms are then discretized with finite elements spaces. By construction at least one minimum of the functional exists, even when the original PDE plus BCs is ill-posed in the sense of Hadamard. Since the discretization is already constructed in a least-squares sense, to add experimental velocity to an incompressible Navier-Stokes simulation we start with a LSFEM functional for N-S, and add a term quadratically penalizing the difference between predictions and observations of local velocities. The weighting of this term determines the extent to which the solution will match the observations and is chosen in inverse proportion to the measurement accuracy.

We show that this formulation is equivalent to finding *Maximum A-Posteriori* (MAP) estimate of a Bayesian updating problem, with a suitably chosen prior and statistical model. The prior is on the full state-vector of the finite-element discretizationEvaluating the likelihood then requires a simple evaluation of the finite-element state at the measurement locations. The Bayesian framework provides added insight, forcing us to explicitly state our statistical assumptions; provides inspiration for modifying the formulation; and guides choice of the weights in the LS functional.

The method is applied to a backward-facing step, with experimental data from Denham and Patrick. The flow and data locations are depicted in Figure 1. Model error comes from coarse-grids, and unknown (and unspecified!) inflow boundary conditions. Example results are seen in Figure 2. With simulation-only (green line) the velocity profile in the channel misses the experimental results (blue dots) completely. By adding small amounts of velocity data at locations either upsteam (black line) or downstream (red line) of the plotted profile, the results match the data precisely. The proposed technique has a cost of a single forward flow simulation. Our method is therefore efficient, statistically justified, and will be shown to be accurate.

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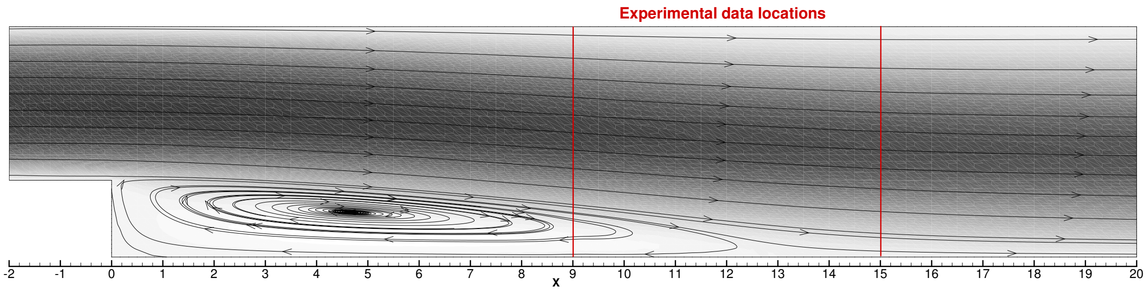

Figure 1: Streamlines and contours of horizontal-flow velocity for the simulation on the fine mesh, with experimental inflow BC, and experimental data inserted at x=9. Alternative data location at x=15 also marked.

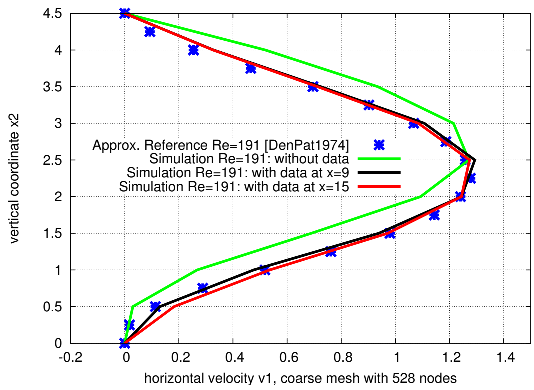

Figure 2: Horizontal velocity profiles. Coarse grid, experimental data at x=9 or x=15.