

# A STABILISED PETROV-GALERKIN FORMULATION FOR LINEAR TETRAHEDRAL ELEMENTS IN COMPRESSIBLE, NEARLY INCOMPRESSIBLE AND TRULY INCOMPRESSIBLE FAST DYNAMICS

Chun Hean Lee\*, Antonio J. Gil, Javier Bonet and Miquel Aguirre

Civil and Computational Engineering Centre, College of Engineering  
Swansea University, Singleton Park, SA2 8PP, United Kingdom

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A mixed second order stabilised Petrov-Galerkin finite element framework for fast solid dynamics was recently introduced [1]. A mixed formulation [2, 3, 4], written as a system of conservation laws for the linear momentum and the deformation gradient, performs extremely well in bending dominated scenarios (even when linear tetrahedral elements are used) yielding equal order of convergence for displacements and stresses (which proves ideal in the case of low order finite elements usually preferred in industry). In this paper, this formulation is further enhanced for nearly and truly incompressible deformations with three key novelties. First, a new conservation law for the Jacobian of the deformation is added into the system providing extra flexibility to the scheme. Second, a variationally consistent Petrov-Galerkin stabilisation methodology is derived. Third, an adapted fractional step method is presented for both incompressible and nearly incompressible materials in the context of nonlinear elastodynamics. The enhanced formulation overcomes locking and non-physical hydrostatic pressure fluctuations in bending dominated nearly or truly incompressible deformations, providing a good balance between accuracy and speed of computation.

## REFERENCES

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