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HIGH ORDER APPROXIMATIONS OF RESERVOIR FLOWS

Jizhou Li*[1] and Beatrice Riviere[2]

[1] Rice University, 6100 Main Street MS 134, Houston, TX 77005, jl48@rice.edu
[2] Rice University, 6100 Main Street MS 134, Houston, TX 77005, riviere@rice.edu

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A high order method in space and in time is formulated for the miscible displacement problem, which is an important process in enhanced oil recovery. The miscible displacement problem is a two-component one-phase problem. Conservation of mass for each component, combined with Darcy's law, yields a system of coupled nonlinear partial differential equations.

We propose a numerical method for solving the miscible displacement problem with discontinuous Galerkin method in space and implicit Runge-Kutta method in time [2]. The method approximates the fluid pressure and the resident fluid concentration. Our algorithm allows us to preserve the high order approximation in both space and time while reducing the computational cost by a decoupling strategy of the pressure and concentration equations.

Our algorithm has been implemented in the DUNE framework [1]. We first show the numerical convergence rates obtained for analytical solutions. The time-step is fixed and the meshes are uniformly refined. Fig. 1 shows the optimal rates in the gradient of the pressure for polynomial degrees varying from 1 to 6 and the corresponding rates for the gradient of the concentration.

Next the method is applied to a quarter-five spot problem. The viscosity of the solvent is 6.9 times larger than the viscosity of the resident fluid. The pressure is approximated by discontinuous piecewise quartics and the concentration by discontinuous piecewise cubics. Fig. 2 shows snapshots of the concentration of the resident fluid at a given time, as well as the velocity fields and the Euclidean norm of the velocity.

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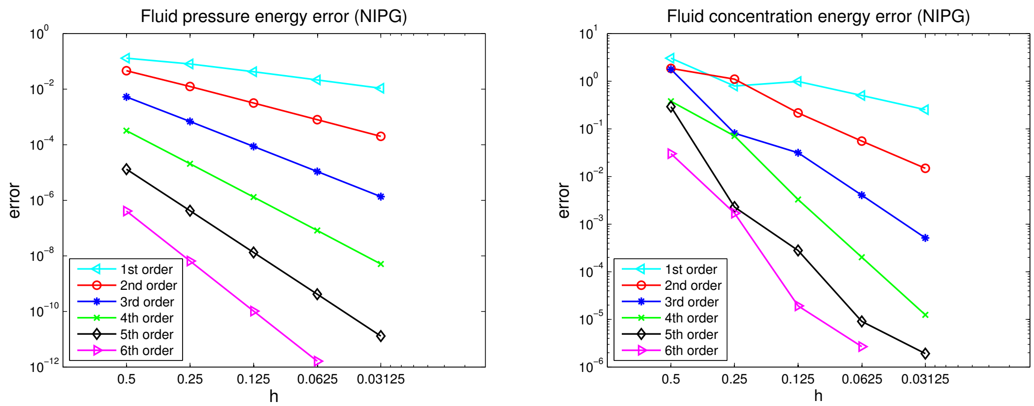

Figure 1: Convergence rate of pressure (left) and concentration (right) in energy norm for various polynomial approximations.

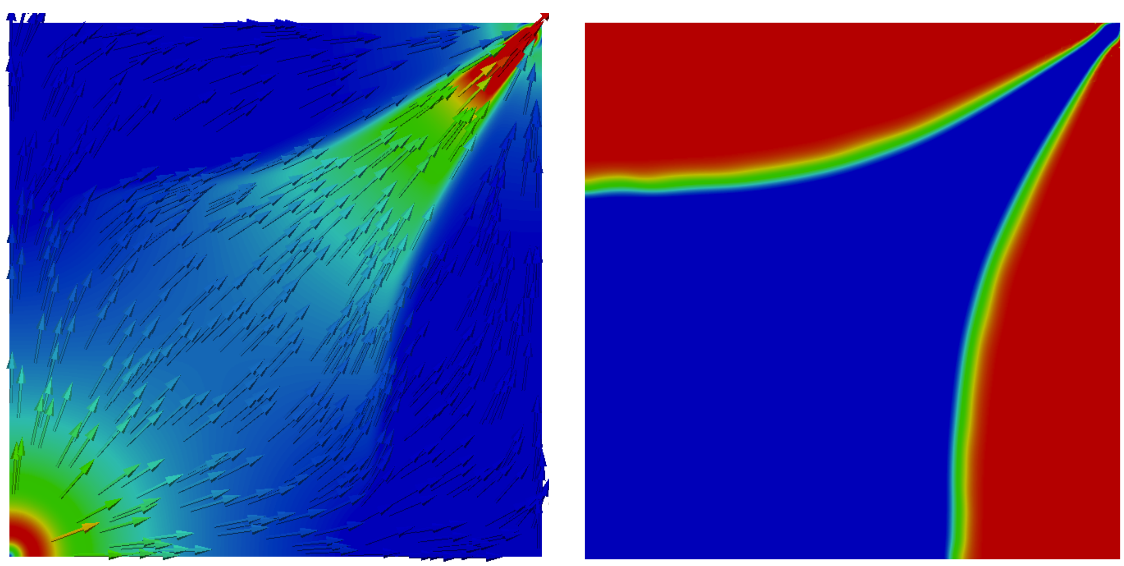

Figure 2: Velocity field and concentration contour of the resident fluid at 15 days. Piecewise cubic polynomials are used.

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