11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

NATURAL CONVECTION AND LOW MACH NUMBER FLOWS SIMULATION USING A COMPRESSIBLE HIGH-ORDER FINITE VOLUME SCHEME

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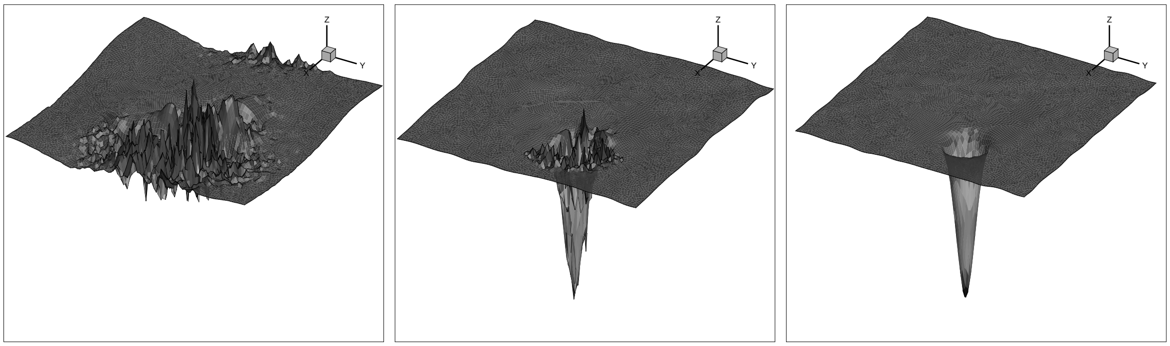
Key words: *High order Finite-Volume, Low Mach number flows*

Modern finite volume codes for compressible flows have to confront the diversity of fluid regimes in many industrial applications. The same code must handle supersonic flows as well as near zero Mach number flows. A specific exemple is aircraft engine design, which gathers several shortcomings of compressible codes: the first one is the so-called *low Mach* number problem, which requires modified flux functions. A second problem is due to the driving term in natural convection. A *well-balanced* treatment of the gravity term is usually required. Furthermore an implicit time-stepping scheme is needed in many practical applications.

The purpose of this talk is to report recent results obtained for various flows in the quasi-incompressible regime using a fourth-order finite volume on general unstructured grids [1] [1]. We consider in Fig. 1 the case of an isothermal Gaussian vortex propagating accross a periodic box. Numerical spurious oscillations in the pressure appear around the vortex. This is typical of the well known *low Mach* number problem with compressible solvers, [2, 3] . The relative magnitude of these oscillations depends on the strength of the vortex. This strength is expressed in terms of the “radial” Mach number $\frac{|u_\theta|}{|c_0|}$. (c_0 is the sound velocity of the background flow). Oscillations are particularly strong in the case of the first-order scheme. As can be observed this behaviour gradually disappears when the reconstruction order increases from first to fourth order.

In this contribution we propose a linear analyis of this phenomenon along the following lines. The semi-discrete FV approximation (first order) of the Euler equations in two

[1] This study takes place within the development of the CEDRE project at ONERA-DSNA



(a) First order: spurious pressure oscillations of the magnitude of the vortex (b) Second order reconstruction: pressure oscillations with 30% of the vortex magnitude (c) Fourth order reconstruction: pressure oscillations are eliminated

Figure 1: Isothermal Gaussian vortex propagating in a square periodic box. The background Mach number is M=0.15. The grid is unstructured. The scheme uses the Roe numerical flux, with implicit time-stepping. 2000 time iterations are performed.

dimensions can be expressed in a periodic Cartesian box as

$$\frac{dW_{i,j}}{dt} + \Delta_x F_{i,j} + \Delta_y G_{i,j} = 0, \quad 0 \leq i, j \leq N-1 \tag{1}$$

where $W = [\rho, \rho\mathbf{u}, \rho e] \in \mathbb{R}^4$ is the conservative variable and $\Delta_x F$ (resp. $\Delta_y G$) is the upwinded flux difference term in the x (resp. y) direction. Linearizing (1) around a constant state W_0 gives a linear vector equation in the pertubation $t \mapsto {W_{i,j}}'(t)$ of the form

$$\frac{d{W_{i,j}}'}{dt} + \left\{ \frac{\partial}{\partial W} \Delta_x F_{i,j}(W_0) + \frac{\partial}{\partial W} \Delta_y G_{i,j}(W_0) \right\} {W_{i,j}}' = 0 \tag{2}$$

In (2) $\frac{\partial}{\partial W} \Delta_x F_{i,j}(W_0)$ (resp.$\frac{\partial}{\partial W} \Delta_y G_{i,j}(W_0)$) stands for the jacobian matrix of the flux difference in the x (resp. y) direction. Due to the periodicity and the linearity, the eigenstructure of (2) can be calculated explicitely, making available the analytical form of the solution of (2). This solution offers a suitable tool for understanding the behaviour of the scheme. In particular the location of the spectrum for several *low Mach* number modified numerical fluxes (such as *Roe*, *HLLC*, *AUSM+*, ...) is investigated: this allows to recover conclusions of previous studies [3]. Finally we will show how the fourth order reconstruction enables to enhance the behaviour of the scheme.

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