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ROBUST NXFEM METHOD FOR A NONCONFORMING APPROXIMATION OF AN ELLIPTIC PROBLEM

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Several finite element methods have been proposed in the last years in order to treat cut meshes. One of them is NXFEM (Nitsche's eXtended Finite Element Method), introduced by A. Hansbo and P. Hansbo in [6] and based on the use of Nitsche's method to treat the transmission conditions on the interface. Some recent developments of NXFEM concern its robustness with respect to the geometry or its application to different model problems such as fluid flow (see for instance [1], [3], [7], [8]).

At our knowledge, the NXFEM method has been used so far with continuous finite element approximations. In particular, most works on Stokes equations employ continuous $(P_2)^2-P_1$ elements for the velocity and pressure, with different stabilisation terms in order to ensure the inf-sup condition.

Our mid-term goal is to extend NXFEM to the case of nonconforming finite elements in order to further apply it to the coupling of Newtonian and non-Newtonian fluids. The application that we have in mind is the modelling of red blood cells in a blood flow. So far, we have considered Darcy and Stokes equations; the next step is to treat the Giesekus model for a viscoelastic fluid, where the use of nonconforming elements has already proved its interest [2], [5].

For P_1-continuous elements the degrees of freedom are associated to the nodes of the triangulation, which belong to only one subdomain; whereas for P_1-nonconforming elements, they are associated to the edges, which can be cut by the interface. So one cannot directly use the natural extension of the interpolation operator introduced in [6] to the nonconforming case. Such an interpolation operator is necessary in order to optimally bound the error, which now contains, besides the usual approximation error, a consistency term due to the nonconformity.

In this talk, we propose two approaches to tackle this problem. We mainly focus on elliptic equations with discontinuous coefficients. We consider triangular meshes and P_1 nonconforming elements of Crouzeix-Raviart [4].

The first approach consists in employing the classical Crouzeix-Raviart elements and in adding some stabilisation terms on each part of the edges cut by the interface, following the discontinuous Galerkin method. The weights used for the definition of the means as well as the stabilisation parameters in each subdomain are chosen in order to get robustness of the method with respect to the geometry. Note that in this case, the additional stabilisation terms compensate the nonconformity error on the cut edges.

In the second one, we modify the basis functions on the cut triangles such that the new degrees of freedom are associated to only one subdomain. No additionnal term is necessary to ensure the stability of the formulation. The difficulty now lies in estimating the interpolation error in a robust way.

Both approaches yield well-posedness of the discrete problem and optimal a priori error estimates. Moreover, we have shown that the first method converges towards the second one when the (numerical) stabilisation parameter tends towards infinity. Numerical tests will be presented in order to validate the previous theoretical results.

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