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ON SHAKEDOWN OF SHAPE MEMORY ALLOYS WITH PERMANENT INELASTICITY

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This communication is concerned with shakedown theorems for Shape Memory Alloys (SMAs). The peculiar properties of SMAs are the result of a solid/solid phase transformation between different crystallographic structures. Much effort has been devoted to developing constitutive laws for describing the behaviour of SMAs. The phase transformation is typically tracked by an internal variable α which - depending on the complexity of the material model - may be scalar or vectorial [5]. In most of SMA models, the internal variable α must comply with some a priori inequalities that result from the mass conservation in the phase transformation process. The presence of such constraints constitutes a crucial difference with standard plasticity models, and calls for special attention when the structural evolution problem is considered. Non-smooth mechanics [4] offer a sound mathematical framework for handling constraints on state variables. The large-time behavior of solids in non-smooth mechanics has been addressed in [6]. A static shakedown theorem has been proposed, taking the form of a sufficient condition for the evolution to become elastic in the large-time limit. When the shakedown limit provided by that theorem is exceeded, it was found that the large-time behaviour is dependent on the initial state: in the case of cyclic loadings, some initial conditions lead to shakedown whereas some others lead to alternating phase transformation. Such a feature is not found in standard plasticity.

The theorem in [6] applies to a wide range of constitutive models of phase transformation in SMAs. Contrary to [3], that theorem is path-independent, in the spirit of the original Melan theorem. Lately, models coupling phase-transformation and plasticity have been proposed in an effort to describe permanent inelasticy effects which are experimentally observed in SMAs [1, 7, 2]: although phase transformation in SMAs is the main inelastic mechanism, dislocation motions also exist and are responsible for such effects as training and degradation in cyclic loadings. To model such a behavior, two internal variables are generally introduced: in addition to a (constrained) variable describing the phase transformation, an additional variable is used to describe plasticity effects. As discussed in [1, 7], it is essential to introduce a coupling term between those two variables in the free energy. Extending the approach used in [6], we present a static shakedown theorem for SMA models coupling phase-transformation and plasticity. For a parametrized loading history, that theorem gives a 'static' safety factor with respect to shakedown. Using min-max duality, a kinematic theorem and a corresponding 'kinematic' safety factor are introduced. A simple example is studied to illustrate the results.

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