

MACROSCOPICALLY CONSISTENT FILTERED ELASTICITY TENSOR FIELDS OF HETEROGENEOUS MEDIA

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Starting from the description at the micro-scale of the elastic properties of an infinite heterogeneous material, a field of non-local stiffness tensor is introduced by filtering the solution to the homogenization problem of the considered heterogeneous material. The filtered stiffness tensor, depending on the filter to heterogeneity size ratio, provides a continuous transition from the actual micro-scale heterogeneous stiffness field to the macro-scale homogenized stiffness tensor. For any intermediate filter size, the homogenization of the filtered stiffness yields exactly the homogenized stiffness, therefore it is called macroscopically consistent. The non-local stiffness tensor is intrinsically non symmetric, but its spatial fluctuations are smoothed, allowing for a less refined discretization in numerical methods.

As a by-product, a two step heterogeneous multiscale method is proposed to reduce memory and computational time requirements of existing direct schemes while controlling the accuracy of the result. The first step is the estimation of the filtered stiffness at sampling points by means of an oversampling strategy to reduce boundary effects. The second step is the numerical homogenization of the obtained sampled filtered stiffness.

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