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A FULLY-NESTED INTERPOLATORY QUADRATURE FOR UNCERTAINTY QUANTIFICATION

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The goal of this study is to alleviate the constraint of the classical 1D interpolatory nested quadratures [3, 2] than one should go from a set of n points to a set of $(2n+1)$ points (for Fejér second rule [5]) or $(2n$-1) points (for Clenshaw-Curtis rule [1, 4]) to benefit from the nesting property. A sequence of recursively included quadrature sets for all odd number of quadrature points is proposed to define interpolatory rules of the classical form,

$$Q[f] \simeq \sum_{i=1}^{n} w_i f(x_i) \quad \text{for the approximation of} \quad I[f] = \int_{-1}^{1} f(x)dx$$

Their accuracy is compared with those of standard rules for very regular functions such that the error can be estimated by

$$|I[j] - Q[f]| \leq \frac{2}{n!} \;\; ||f^{(n)}||_\infty \;\; ||\prod_{l=1}^{l=n}(x - x_l)||_\infty,$$

and also for continuous functions where the error is expressed as

$$|I[f] - Q[f]| \leq 2E_{n-1} + \sum_{i=1}^{i=n} |w_i| E_{n-1},$$

where E_{n-1} is the supremum norm of the difference between f and its best approximation in this sense by a polynomial of degree $(n-1)$. In particular, all weights of the sets are proved to be positive for low numbers of quadrature points, which is the optimal case in the sense of the previous error bound.
The corresponding quadrature rules are applied to classical mathematical tests for numerical integration and then to an uncertainty quantification problem based on a three dimensional external flow. The actual convergence towards limiting integral values when increasing the sample size n is studied for the proposed quadrature compared to Gaussian quadrature.

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