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ON GLOBAL-LOCAL ENRICHMENTS FOR EVOLUTION EQUATIONS

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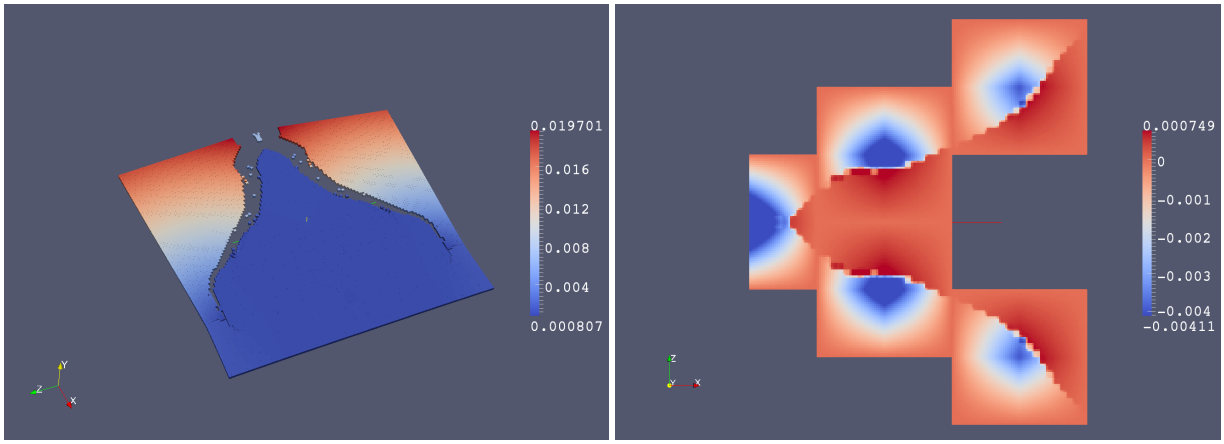
Generalized Finite Element Methods obtain their approximation power through the use of local, problem-dependent enrichment functions. Whenever these are not available a priori through e.g. asymptotic expansions of the solution, computing these from local problems might be a worthwhile approach.

In the Finite Element world one way of extracting local problems to be used for improving a coarser solution is via the Iterative Global-Local approach [1, 2]. Fine-scaled local problems with corresponding boundary conditions are extracted from a coarser-scaled global solution. Then, the solutions of these local problems are used in the computation of the residuals for the global solution in an iterative fashion.

In the Generalized Finite Element world this idea has been taken up by, among others, Duarte et al. [3, 4]. Here, the information transfer from the finer to the coarser scale is now done via the use of the local solutions as local enrichments. However, iterations of the procedure might be necessary since the boundary conditions used for computation of the local solutions might not align with the enriched solution.

These papers focus on static or quasi-static problems. Moreover, the underlying Partition of Unity are always traditional Lagrangian Finite Elements.

We extend some of this prior work to encompass a flat-top Partition of Unity as used in the Particle-Partition of Unity [5, 6] flavor of Generalized Finite Elements. Furthermore, we focus on time dependent problems. Thus, the boundary conditions of the local problem will be the evolution on the boundary. We discuss various strategies for the extraction of these from the global solution. Additionally, we give some results on the interplay of the Global-Local spatial discretization and time integration.



(a) Discontinuous solution from local particle simulation

(b) 5 resulting new shape functions for global solution

Moreover we also discuss the use of explicit particle simulations to obtain solutions to the local problems [7]. We address issues of constructing discontinuous enrichments from discrete displacement fields with incidence information. Regarding these, we present some numerical results from using Peridynamics for local problems.

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