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WANG CUBES IN NUMERICAL HOMOGENIZATION METHODS

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As has been shown in [1], employing the concept of Wang tilings in microstructure compression and reconstruction techniques surpasses the popular and widely used concept of Statistically Equivalent Periodic Unit Cell, namely in the sense of reducing long-range order spatial artefacts. In this contribution we bring this concept in three dimensions considering Wang cubes [2] instead of planar tiles as basic elements of tilings.

The concept of Wang tilings, originally introduced by Hao Wang [3], is based on a finite set of square dominoe-like tiles with codes assigned to the tiles edges. Placing these elements, members of specific sets, side by side such that the congruent edges share the same code allows to produce tilings, so microstructure domains, of arbitrary sizes.

An attractive feature of the tiling concept from the Materials Engineering point of view is its ability to synthesize a non-periodic computational domain that resemble geometric features of reference media. In this contribution we employ the 3D stochastic tiling algorithm based on propositions due to Cohen et al. [4].

The desired morphology of Wang cubes is designed in a way to meet spatial features of a reference medium by employing optimization procedures and objectives given by the two-point probability function. Hence it allows for a very efficient investigation of a large number of Representative Volume Elements (RVEs) of different sizes as requested e.g. in [5].

The stiffness matrix of an RVE is assembled from Schur complements of stiffness matrices of individual cubes. The internal degrees of freedom of each cube are eliminated and only their boundary counterpart is taken into account. Therefore, the number of unknowns in an RVE is significantly reduced. As a consequence, a finer mesh can be used on the level of cubes which leads to higher accuracy. The method benefits from the repetitive nature of a tiling, in other words, a regular lattice made up of a limited number of cube templates placed in a repetitive manner.

Without loss of generality, the scope of this work is limited to the overall thermal conductivity of the random monodisperse medium consisting of nonpermeable spheres.

Moreover we believe that the present work may draw potential attention also from the Generalized Finite Element community as the Wang tilings can be as well as used for the assembly of microstructure-informed enrichment functions [7].

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REFERENCES

[1] J. Novák and A. Kučerová and J. Zeman. Compressing random microstructures via stochastic Wang tilings. *Phys. Rev. E*, Vol. **86** (4), 040104, 2012.

[2] K. Culik II and J. Kari. An aperiodic set of Wang cubes. *J. Univers. Comput. Sci.*, Vol. **1** (10), 675–686, 1995.

[3] H. Wang. Proving theorems by pattern recognition-II. *Bell Syst. Tech. J.*, Vol. **40** (1), 1–42, 1961.

[4] M. F. Cohen and J. Shade and S. Hiller and O. Deussen. Wang tiles for image and texture generation. *ACM T. Graphic.*, Vol. **22** (3), 287–294, 2001.

[5] J. Dirrenberger and S. Forrest and D. Jeulin. Towards gigantic RVE sizes for 3D stochastic fibrous networks. *Int. J. Solids Struct.*, Vol. **51**, 359–376, 2013.

[6] J. C. Michel and H. Moulinec and P. Suquet. Effective properties of composite materials with periodic microstructure: a computational approach. *Comput. Methods Appl. Mech. Engrg.*, Vol. **172**, 109–143, 1999.

[7] J. Novák and A. Kučerová and J. Zeman. Microstructural enrichment functions based on stochastic Wang tilings. *Modelling Simul. Mater. Sci. Eng.*, Vol. **21** (2), 025014, 2013.