11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

UNFITTED NITSCHE FINITE ELEMENT METHODS FOR MULTI-PHYSICS PROBLEMS

S. Claus[1,*], A. Massing[2] and E. Burman[1]

[1] Department of Mathematics, University College London, Gower Street, London WC1E 6BT, susanne.claus@ucl.ac.uk

[2] Center for Biomedical Computing at Simula Research Laboratory, P.O. Box 134, 1325 Lysaker, massing@simula.no

Key words: *Nitsche Methods, Multi-physics Problems, a Priori Error Analysis, Three Field Stokes.*

In this presentation, we will discuss the application of Nitsche type coupling conditions to systems with different physical properties that are coupled across an interface. In the presented method, the interface between the different systems may cut through elements of the finite element mesh and the approximation may be discontinuous inside an element. This has the advantage that problems involving large topological domain changes become more computationally feasible in contrast to methods in which the mesh has to be fitted, moved and regenerated. However, in order to restore the accuracy and stability of the unfitted method independent of the interface location stabilisation techniques have to be employed and the underlying finite element discretisation has to be enriched. We have developed stabilisation techniques tailored to Nitsche type coupling conditions that restore the stability and accuracy independent of how the interface cuts the mesh for a variety of different PDE systems.

Nitsche's method has several advantages over alternative methods that can be used to enforce weak interface coupling conditions, such as Lagrange multiplier or penalty methods. In contrast to classical Lagrange multiplier methods no other unknown is introduced in the Nitsche method and no discrete inf-sup condition needs to be satisfied. And conversely to standard penalty parameter techniques, the resulting Nitsche method is consistent [1]. However, the interface boundary may cut the computational mesh in such a way that some of the resulting subelements belonging to different materials become very small and the linear system arising from Nitsche's discrete problem may be ill-posed [2]. To overcome this difficulty, we suggest the addition of ghost-penalty terms to the variational formulation over the band of elements that are cut by the interface [2, 3].

In this contribution, we will first discuss the development and application of a stable and accurate method for three field Stokes' problems, both in the case of multi-physics coupling across the interface and in the case of the fictitious domain method. Furthermore, we will present a scheme for a falling rigid particle in a fluid. The implementation of the method is realised in the finite element software package FEniCS [4].

REFERENCES

[1] P. Hansbo. Nitsche's method for interface problems in computational mechanics. *GAMM-Mitt.*, Vol. **28**(2), 183–206, 2005.

[2] E. Burman. Ghost penalty. *Comptes Rendus Mathematique*, Vol. **348** (21-22), 1217 – 1220, 2010.

[3] E. Burman and P. Zunino. Numerical Approximation of Large Contrast Problems with the Unfitted Nitsche Method. *Frontiers in Numerical Analysis - Durham 2010, Lecture Notes in Computational Science and Engineering*, Vol. **85**, 227–282, 2012.

[4] A. Logg, K. Mardal, G.N. Wells et al. *Automated Solution of Differential Equations by the Finite Element Method.* Springer, 2012.