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ON THE SENSITIVITY OF THE POD TECHNIQUE FOR FLUID FLOWS AND FLUID-STRUTURE INTERACTION PROBLEMS

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1 Introduction

Models reduction techniques are widely used for problems involving parametric evolution, like the optimal control problems for parameters estimate [Allery et al., CNSNS, 2005], [Allery et al., IAM, 2008] and [Jorres et al., COA, 2013], the Fluid-Structure Interaction problems [Balajewicz et al., BAPS, 2013], [Liberge et al., JFS, 2010] and [Liberge et al., EJCM, 2007], stability study [Lassila et al., MMNA, 2012], form optimisation problems [Manzoni et al., IJNMF, 2012] and [Lassila et al., CMAME, 2010], etc... We are interested in mathematical a priori error upper bounds of an original parametric ROM by POD. An original POD basis is considered : It is associated with a reference solution corresponding to a reference value of the parameter, and with its parametric derivative at the same reference parameter value. A mathematical a priori estimate of the parametric squared L^2-error induced by the corresponding ROM is developed. This result is an improvement of the one we developed in [Akkari et al., JCAM, 2013], where we showed an a priori estimate of the parametric squared L^2-error induced after the model reduction of single parameterized quasi-linear parabolic problems, by a reference POD basis associated with a reference solution. For simplicity reasons, we will present our result in this paper only for fluid flows, but other examples will be shown during the conference for academic applications in Fluid-Structure Interaction (FSI).

2 Mathematical formulation and Main results

We denote $X = [L^2(\Omega)]^d$, $V = [H^1(\Omega)]^d$ where Ω is a bounded open set, connected and lipschitz of $\mathbb{R}^d$, where $d = 2$ or 3. We suppose that our parabolic quasi-nonlinear problem is written formally as follows :

$$\begin{cases} \dfrac{d}{dt}(u_\lambda(t), v)_X = (A(u_\lambda(t); \lambda), v)_X \quad \forall v \in V \\ (u_\lambda(0), v)_X = (u^0_\lambda, v)_X \quad \forall v \in V \end{cases}, \tag{1}$$

(.) denotes the scalar product in the corresponding space. And, $\lambda \in \mathbb{R}^{+*}$ denotes the parameter of these equations. We denote λ_0 an arbitrary parameter value. Let $\Phi^{\lambda_0} = \left(\Phi_n^{\lambda_0}\right)_{n\geq 1}$ be a POD basis in X, associated with u_{λ_0} and $\dfrac{\partial u_\lambda}{\partial \lambda}(\lambda_0)$ on a time interval $(0, T)$. Then, the associated reduced solution is given by :

$$\begin{cases} \dfrac{d}{dt}\left(\hat{u}_{\lambda,\lambda_0}(t), \Phi_n^{\lambda_0}\right)_X = \left(A(\hat{u}_{\lambda,\lambda_0}(t);\lambda), \Phi_n^{\lambda_0}\right)_X \\ \left(\hat{u}_{\lambda,\lambda_0}(t), \Phi_n^{\lambda_0}\right)_X = \left(u_\lambda^0, \Phi_n^{\lambda_0}\right) \end{cases} \tag{2}$$

Then, the question which arizes naturally is the following : To what extent $\hat{u}_{\lambda,\lambda_0}$ remains an accurate approximation to u_λ ?

Formal result 1

$$\|u_\lambda - \hat{u}_{\lambda,\lambda_0}\|^2_{L^2(0,T;X)} \leq \sum_{n=N+1}^{\infty} \mu_n^{\lambda_0} + 4T \sum_{n=N+1}^{\infty} \mu_n^{\lambda_0} \left|\lambda - \lambda_0\right|^2 + \left|\lambda - \lambda_0\right|^4 \tag{3}$$

3 Numerical experiments

In this paper we present a numerical experiment of result 1 for an incompressible 2D fluid flow in a channel around a circular cylinder (figure 1). The previous parameter λ denotes here the viscosity. We consider a reference parameter value λ_0 corresponding to a Reynolds number equal to 100, and we varied λ in such a way to have computations associated with Reynolds numbers in the interval $[70, 180]$.

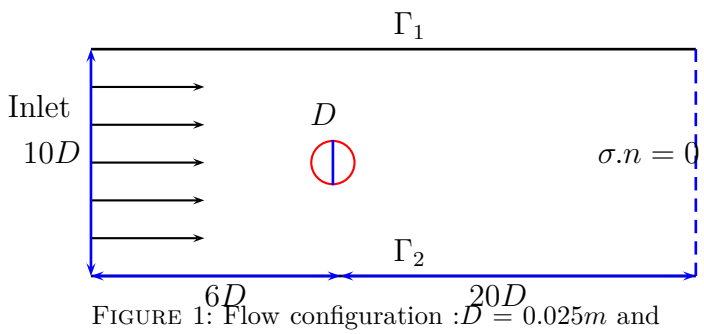

FIGURE 1: Flow configuration :$D = 0.025m$ and the inlet condition is a flat fluid flow of which velocity is : $U_0 = 0.004m\mathring{u}s^{-1}$

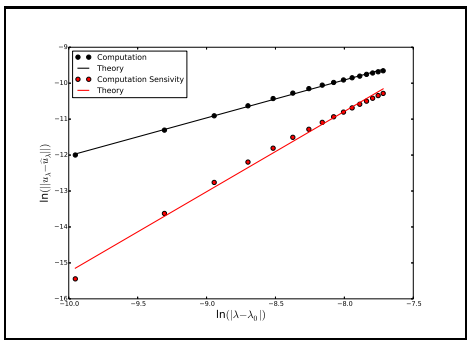

FIGURE 2: Red line : The logarithm of the error relative to the model reduction by our original POD basis. Black line : The logarithm of the error relative to the model reduction by a reference POD basis

4 Conclusion and Prospects

Result 1 shows a strength of the reduced order model. Indeed, figure (2) shows that the case for which the parametric sensitivity of the flow is also included in the snapshots set, is better than the one where we had only snapshots associated with a reference solution : The slope of the red line is clearly stronger than the black one. Which improves our result in [Akkari et al., JCAM, 2013]. Other examples will be shown during the conference in the same aim, but for academic applications in Fluid-Structure Interaction.