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PROPAGATION OF WAVES IN INFINITE BEAMS: PML APPROACH

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Abstract. The problem of transverse vibration of an infinite beam on elastic foundation involves the elastodynamic wave equation over an unbounded domain. This paper deals with such a semi-infinite Bernoulli-Euler beam subjected to a harmonically varying concentrated load at its origin. A common application of this beam is in the investigation of railroad tracks. Since modelling of unbounded domains with radiation damping requires absorption of the waves, the Perfectly Matched Layer (PML) approach, which provides an appropriate solution technique for such problems, is employed here. This method has been widely used for electromagnetic wave problems. It has also been utilized for some dynamic soil-structure-interactions as well as for simulation of earthquake ground motions.
For the general case of infinite domains with inhomogeneous media application of the initial and boundary conditions is difficult to effect in closed form. In addition, irregularities in geometry, material, and viscoelasticity of the foundation will necessitate a numerical procedure. The PML technique along with finite element discretization can offer a suitable means of dealing with such problems. In this manner the response is carried out by dividing the domain into two segments; a bounded portion and an artificial, wave absorbing one. This framework is implemented with a displacement-based Finite Element Method (FEM) in the context of a Galerkin scheme. The first step in the PML process is to transform the governing equations into frequency domain. In this domain the spatial variable is stretched, in complex coordinates, resulting in damping of the outgoing waves. The decaying function used for this purpose is selected in such a manner as to prevent reflecting waves.

Most PML applications so far have dealt with lower order governing differential equations. The case of beams on elastic foundation, considered here, involves a fourth-order equation. The process starts out by first reducing the fourth order equation into four first-order equations. The auxiliary variables thus introduced lead to internal moments and shear forces. The governing equations are then transformed back into the time domain. A weak form of the governing equations is spatially discretized by the use of standard FEM. This leads to the final form of the governing equations of the system. The latter equations are solved by a step-by-step algorithm,

carried out in MATLAB environment. The efficiency and accuracy of the results obtained by the method described here are validated by comparing the results to existing regular numerical solutions for some simple problems. Parametric studies are conducted on the effect of various PML parameters and of the stretching functions in order to establish the best form of these parameters.

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