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MAIN ISSUES IN ANISOTROPIC MESH ADAPTIVE FMG

G. Brethes[1], O. Allain[2], A. Dervieux[1]

[1] INRIA Sophia Antipolis, 2004 route de Lucioles, 06902 Sophia-Antipolis, France,
Gautier.Brethes@inria.fr,
http://www-sop.inria.fr/members/Gautier.Brethes/Gautier.Brethes.htm
[2] LEMMA - 2000 route des Lucioles - Les Algorithmes - Bt. Le Thales A, Biot, 06410,
France,olivier.allain@lemma-ing.com
[3] INRIA Sophia Antipolis, 2004 route de Lucioles, 06902 Sophia-Antipolis, France,
Alain.Dervieux@inria.fr

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Rather earlier in the development of multi-grid (MG) methods, the question of the combination of MG with mesh adaption was risen. See for example [1]. The proposed communication adresses several issues related to the building of a full-multi-grid (FMG) method in combination with an anisotropic, metric-based mesh adaption. We first discuss the building of an anisotropic adaptive multi-grid method and then address the extension to FMG.

Adaptive MG. MG cycling uses several given meshes and advances to convergence the unknown solution w of the discrete PDE. In anisotropic mesh adaption, the unknown is now the couple $(\mathcal{M}, w)$ of the PDE unknown and of a metric $\mathcal{M}$, *i.e.* a field defined on the computational domain Ω and such that on any $\mathbf{x}$ of Ω, $\mathcal{M}(\mathbf{x})$ is a $d \times d$ symmetric definite positive matrix (d dimension of space). A natural approach is to devote the MG cycling to finding w while $\mathcal{M}$ will be in turn derived from w. Therefore the MG cycling will be defined as an internal loop inside the mesh adaptive one. It remains to explain how the different levels necessary for MG are derived.

A metric for several level: The metric is diagonalised as follows (2D):

$$\mathcal{M}(\mathbf{x}) = (h_1(\mathbf{x})h_2(\mathbf{x}))^{-1}\,\mathcal{R}(\mathbf{x}) \begin{pmatrix} h_2(\mathbf{x})/h_1(\mathbf{x}) & \\ & h_1(\mathbf{x})/h_2(\mathbf{x}) \end{pmatrix} {}^t\mathcal{R}(\mathbf{x}) \qquad (1)$$

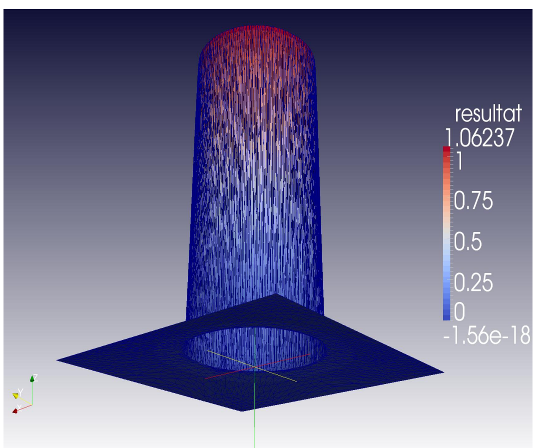
where $h_1(\mathbf{x}), h_2(\mathbf{x})$ are mesh sizes in principal directions. Let $h_1 < h_2$. An anisotropic coarsening is easily defined by putting: $h_1^C = 2h_1$, $h_2^C = Min(h_1^C, 2h_2)$. This strategy is sufficient for many CFD applications.

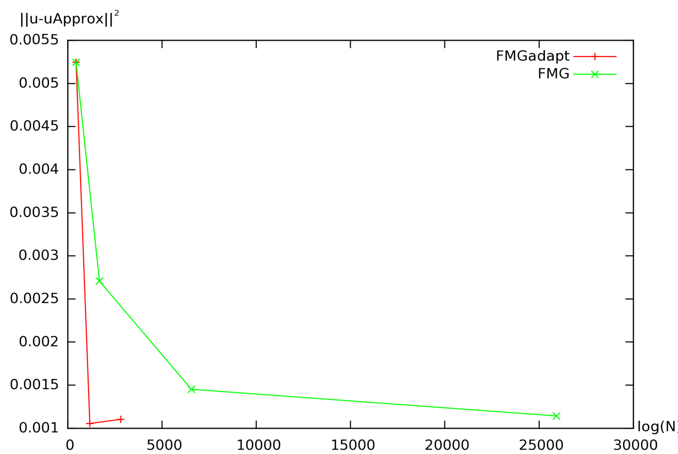
MG convergence: An important issue in MG efficiency is the use of a rather accurate stopping criterion, in order not to waste useless cycles. In our proposal, cycling is stopped from a comparison between the iterative residual and an *a posteriori* local error estimate.

Adaption loop: The external adaption loop needs to update the metric and generate new grids. In the proposed communication, we compare two strategies. First a rather standard Hessian-based adaptation is applied. A second approach uses a norm-oriented adaptation extending to the minimum approximation error norm the so-called goal-oriented adaptation.

Adaptive FMG. The principle of FMG lies in the synergy between successive refinement and MG mesh independent convergence. While MG shows in good cases a $O(NLog(N))$ complexity, FMG shows -still in good cases- $O(N)$ complexity, as soon as the succession of meshes used gives the asymptotic convergence, see for example [2]. This of course cannot happen for the coarsest meshes of the sequence. It can also fail for the finer ones, as soon as small scales of the solution are not captured by a part of these meshes. If each FMG phase is stopped after a fixed residual reduction, the result will be wrong. Conversely, if a convenient stopping criterion is used for each phase, the result will be good, but the overall complexity will be out of control, higher of the promised $O(N)$. As a method for producing asymptotic convergence, we propose anisotropic mesh adaption.

Numerical experiment. The above methods are applied to the solution of a Poisson problem giving the pressure of capillary flow (Fig.1). We compare in Fig.2 the accuracy obtained by non-adaptive FMG and anisotropic adaptive FMG for the same number of nodes. For similar accuracy the gain in CPU is more than a factor 6.





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