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COMPUTATIONAL HOMOGENIZATION FOR TRANSIENT PHENOMENA IN HETEROGENOUS MATERIALS

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In recent years, significant progress has been achieved in the development of the so-called metamaterials, exhibiting 'exotic' properties, usually not existent in natural materials [1]. One of the examples are the acoustic metamaterials designed to attenuate sound propagation at the structural level for certain frequencies. The unique features of these metamaterials originate from the complex interaction of transient phenomena at the microscopic and macroscopic scales, with local resonance occurring within (one of the) micro-constituents resulting in effective band gaps at the macroscale.

Up to date, most of the analytical and numerical approaches used to analyse the behaviour of these materials relied on the linearity assumption, e.g. asymptotic homogenization or multiple scattering theory of elastic waves based on the linear Helmholtz equations. Moreover, these techniques are usually suitable for the analysis of the effective dynamic properties of an infinite material, without taking into account the response of a structure made thereof, which may change the local transient phenomena. In addition, it is expected that exploiting the non-linear regimes may open even broader possibilities in the design of acoustic metamaterials. As metamaterials will be used increasingly in a near future, it will be necessary to study their ultimate properties, e.g. inelastic behavior, damage, fatigue, etc. A powerful technique to bridge the microstructural behaviour and macroscale response, particularly suitable for complex microstructures with non-linear behaviour and evolution, is the so-called computational homogenization (FE^2), see e.g. [2], among many others. Classical (first-order) computational homogenization schemes require, however, full separation of length and time scales, which clearly does not hold for locally resonant metamaterials.

In this work, a novel transient computational homogenization procedure that is suitable for the modelling of the evolution in space and in time of materials with non-steady state microstructure, such as metamaterials, is presented [3]. This transient scheme is an extension of the classical (first-order) computational homogenization framework. In the new approach the separation of scales hypothesis is relaxed that allows going beyond the long wavelength approximation for the microstructural components. It is based on an enriched description of the micro-macro kinematics by allowing large spatial fluctuations of the microscopic displacement field compared to the macroscopic displacement field, arising from the local resonance phenomena. From the microstructural analysis, the macroscopic stress and the macroscopic linear momentum are obtained from an extended Hill-Mandel macrohomogeneity condition. In particular, the full balance of linear momentum is solved at both scales.

Consistent time discretization towards the numerical implementation of the framework, as well as a formulation specialized for linear elasticity, are provided to ensure a more efficient calculation of the effective response. As an example, the transient computational homogenization approach is applied to the multi-scale analysis of a metamaterial subjected to dynamic uniaxial loading. Consistency of the framework with respect to Direct Numerical Simulations is shown for a large range of loading frequencies and microstructural sizes. Attenuation of the macroscopic waves at specific loading frequencies, as well as size effects, are highlighted.

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