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IMPROVEMENTS ON THE NUMERICAL ANALYSIS OF VISCOPLASTIC-TYPE NON-NEWTONIAN FLUID FLOWS

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The aim of this work is to delve into the numerical analysis of viscoplastic-type non-Newtonian fluid flows (see [1, 2, 3], for instance) with the objective of carrying out more advanced numerical simulations for them. Specifically, improvements in the spatial discretization schemes and the temporal integration methods are proposed to overcome the numerical problems introduced by the transpose diffusive term and associated with the velocity field discontinuity, the artificial viscous dissipation and the transpose viscous coupling.

Concerning the velocity field discontinuity, the discretised transpose diffusive term should be composed of contiguous values of the collocated discrete variable in order to reproduce faithfully the non-Newtonian behaviour. An expression for the transpose diffusive term based on the divergence theorem and with the staggered velocity gradient approached by an arithmetic mean (AM) is proposed:

$$\nabla_h \cdot (\eta(\nabla_h \boldsymbol{v})^T) = \frac{1}{\Omega_P}\left(\frac{1}{2\,\Omega_P}\sum_{PF}\boldsymbol{A}_{PF}\frac{\boldsymbol{v}_F}{2}\right)\cdot\sum_f \eta_f \boldsymbol{A}_f + \frac{1}{\Omega_P}\sum_f\left(\frac{\eta_f}{2\,\Omega_F}\sum_{F2F}\boldsymbol{A}_{F2F}\frac{\boldsymbol{v}_{2F}}{2}\right)\cdot\boldsymbol{A}_f \tag{1}$$

As for the artificial viscous dissipation, the aforementioned term should be cancelled when the non-Newtonian viscosity takes the value of the Newtonian viscosity under the hypothesis of incompressible fluid (see Figure 1a), this time with the objective of reproducing accurately the Newtonian behaviour. Another expression for the transpose diffusive term based on the Green's first identity is proposed:

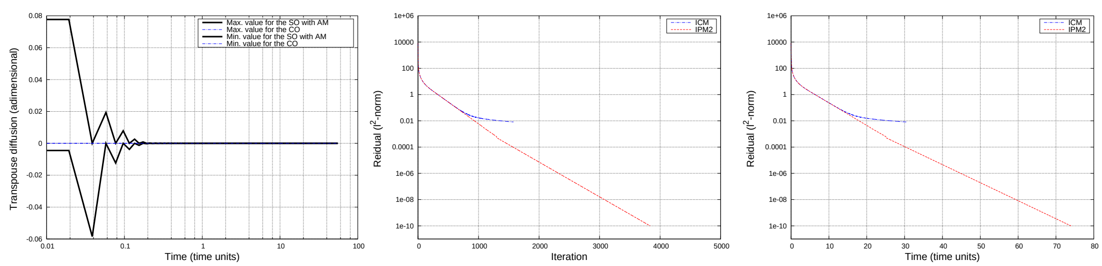
$$\nabla_h \cdot (\eta(\nabla_h \boldsymbol{v})^T) = \frac{1}{\Omega_P}\left[\left(\frac{1}{\Omega_P}\sum_{PF}\boldsymbol{A}_{PF}\,\frac{\boldsymbol{v}_F}{2}\right)\cdot\left(\sum_f \eta_f\,\boldsymbol{A}_f\right)\right] \tag{2}$$

Regarding the transpose viscous coupling, the transpose diffusive term has been analysed in the field of the temporal integration methods (advanced, projected and corrected step). For the viscoplastic-type non-Newtonian fluid flows with a permanent regime and a steady state, the implicit projection method with a special treatment for the pressure term (IPM2) has turned out to be robust and accurate while presents an efficiency that appears to be much closer to the optimum than in the others analysed methods, see Figures 1b and 1c. The temporal discretization scheme of the pressure term has proved to be the key aspect for the aforementioned method.

$$\frac{(\rho \boldsymbol{v})_c^{n*+1} - (\rho \boldsymbol{v})_c^{n}}{\Delta t} = -C(\boldsymbol{v}_f^{n*+1})\boldsymbol{v}_c^{n*+1} + D\boldsymbol{v}_c^{n*+1} + D^T\boldsymbol{v}_c^{n*+1} - Gp_c^n \tag{3}$$

$$\frac{(\rho \widetilde{\boldsymbol{v}})_c^{n*+1} - (\rho \boldsymbol{v})_c^{n*+1}}{\Delta t} = \frac{1}{2}Gp_c^n \qquad \frac{(\rho \boldsymbol{v})_c^{n+1} - (\rho \widetilde{\boldsymbol{v}})_c^{n*+1}}{\Delta t} = -\frac{1}{2}Gp_c^{n+1} \tag{4}$$

$$\frac{(\rho \boldsymbol{v})_c^{n+1} - (\rho \boldsymbol{v})_c^{n*+1}}{\Delta t} = -\frac{1}{2}\Delta t G\left(\frac{p_c^{n+1} - p_c^n}{\Delta t}\right) \tag{5}$$



(a) Transpose diffusion (b) Convergence history (c) Temporal evolution

Figure 1: Some numerical improvements

To confirm and extend the current results are part of our current and future research plans.

REFERENCES

[1] M. M. Molla and L. S. Yao. The flow of non-newtonian fluids on a flat plate with a uniform heat flux. *Journal of Heat Transfer*, 131(1):011702, 2009.

[2] M. M. Molla and L. S. Yao. Non-newtonian natural convection along a vertical heated wavy surface using a modified power-law viscosity model. *Journal of Heat Transfer*, 131(1):012501, 2009.

[3] M. M. Molla, A. Hossain, B.-C. Wang, and D. C. S. Kuhn. Large-eddy simulation of pulsatile non-newtonian flow in a constricted channel. *Progress in Computational Fluid Dynamics*, 12:231–242, 2012.