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ON SOME ASPECTS OF A POSTERIORI ERROR ESTIMATION IN THE MULTIPOINT MESHLESS FDM

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Reliable error estimation of any numerical solution of a boundary value problem constitutes an important part of such analysis. As the exact solution is unknown, the true error of the obtained rough solution is evaluated by using an improved (e.g. high order approximation) solution as the reference one. Among various methods providing the higher quality results, the new multipoint meshless finite difference method (MFDM) [3, 4] may be also used.

The multipoint approach is based on arbitrary irregular meshes, the moving weighted least squares (MWLS) approximation [2] and the global, local, or global-local formulations of b.v. problems. The objective of this work is to formulate and provide an a posteriori error estimation of the global or local solution and residual errors based on the multipoint MFDM.

A posteriori error analysis [1] of the multipoint MFDM results may be applied for two purposes: to examine the solution quality and to generate series of adaptive meshes. Several types of error estimations are currently available [7]. Among them:

- *Hierarchic estimators* – based on the comparison of calculated results with reference solution obtained using *h*-, *p*-, or *hp*- approach. The multipoint method may be successfully applied to this purpos.
- *Residual estimators* – use either explicit residual errors or equivalent implicit ones (not specified here). Each of them provides a quality measure of the higher (multipoint approach) or lower (standard MFDM) order solution error.
  Using approximated higher order solution defined at the nodes, one may calculate the residuum between the nodes. At the middle of this distance, the residuum is expected to reach its maximum value. However, the 1D tests done showed that the error distribution essentially depends on the smoothing parameter g used in the MWLS (Fig. 1).
- *Smoothing estimators* (well known as Zienkiewicz-Zhu one) – based on the comparison between the recovered and the reference derivatives (e.g. stresses). Using multipoint method one may also obtain higher order approximation of derivatives up to order p and use them instead of the recovered ones. It is worth noticing here, that in multipoint approach the convergence rate for solution is of the same order as for its derivatives.

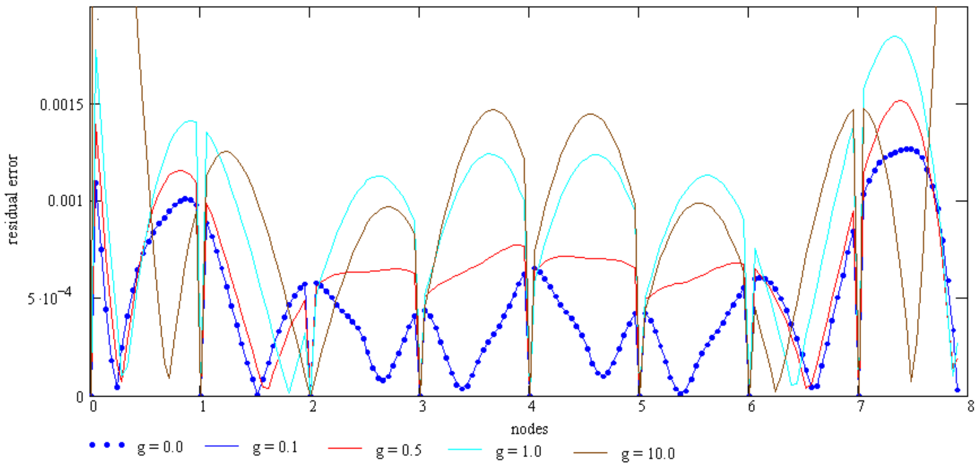

Fig.1: Residual error distribution using 20 points between nodes. The influence of the MWLS weight factor *g*

Due to high quality of its results, the multipoint method may be also used to develop reference solutions needed for a posteriori error (global and local) estimation. A posteriori error analysis of the multipoint MFDM may be applied for two purposes: examination of the solution quality and generation of a series of adaptive meshes. Multiple preliminary tests confirm high quality of a posteriori error estimation based on the multipoint MFDM.

Further development of the use of the multipoint MFDM a priori and a posteriori error analysis is planned.

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