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NON-LINEAR VIBRATIONS OF ROTATING CANTILEVER BEAMS: FINITE ELEMENTS VALIDATIONS OF VARIOUS REDUCED ORDER MODELS

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The large amplitude non-linear vibratory behavior of a rotating cantilever beam is addressed in this study. The motivation of this work is to built a simplified model for analyzes of turbomachinery and turbopropeller blades vibrations in the geometrically non-linear regime. Since blades are designed more and more flexible, in particular when composite materials are used, quantifying the amount of non-linear effects on the vibratory characteristics such as the resonance frequencies and predicting possible jump phenomena is essential. Moreover, we also want to address the effect on those non-linearities of centrifugal forces due to rotation.

In this study, three models are compared. The beam vibrations are restricted to the plane that contains the rotation axis and the beam at rest. The first model (denoted as VK) is based on the classical von Kármán assumptions, that keeps in the strain-displacement law only the first non-linear flexural term in the axial strain. It successfully predicts the non-linear axial/flexion coupling that arises in plates and shells [7] as well as in beams with axially fixed ends. In the case of cantilever beams, the VK model gives a linear response because the non-linearities are due to curvature and appear at a higher order. However, when centrifugal forces are applied to the beam, the von-Kármán model predicts non-linear behaviors [6, 1]. The second model (denoted as Inxt.)is an original analytical model based on an inextensionality constraint has been proposed in the literature, [3]. This model has been widely used to predict non-linear vibrations of non-rotating cantilever beams [5]. This work extends it to the rotating case.Finally, the third model (denoted as FE) is based on a finite-elements discretization of the beam geometry, using a non-linear dynamic formulation of a total Lagrangian Timoshenko plane beam element. Whereas VK and Inxt. models are valid for moderate rotations, the FE model enables large amplitude simulation with no restrictions on the rotation amplitude and thus constitute a reference solution.

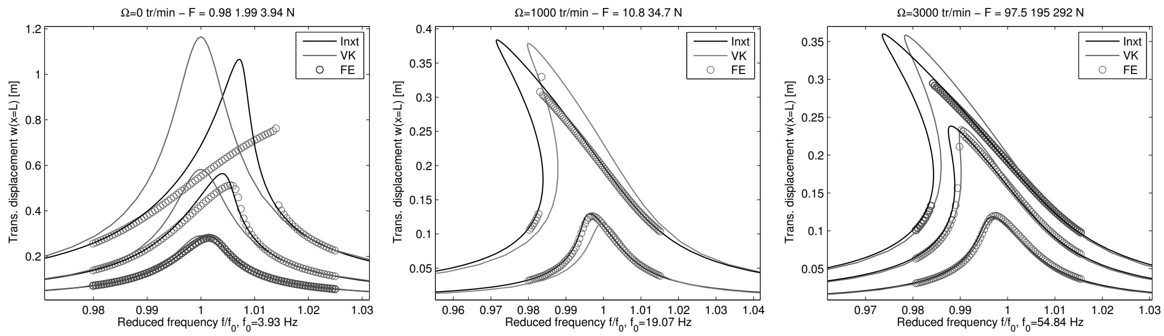

Figure 1: Resonance curves for mode 1 at various rotation speed. Comparison of results from Inxt., VK and FE (with upward frequency sweep) models

The VK and Inxt. models, based on a partial differential equation, are discretized using the vibration mode basis of the associated *linear* and *non-rotating* (i.e. without the centrifugal prestress due to rotation) problem and are solved by a continuation method (HB/ANM [2, 4]). Results for the FE model are obtained by numerically integrating it in time, with a Newmark scheme coupled to a Newton-Raphson algorithm at each time step.

The beam vibratory behavior, stemming from each of the three models, is compared on fig 1. The qualitative behavior of each beam's mode is obtained: the first mode is hardening for low rotation speed and becomes softening for large ones. One can show that the centrifugal force, due to rotation, has a softening effect. The validity range of VK and Inxt. models can also be deduced, by comparison with the FE model. The VK, inaccurate for low rotation speeds, becomes closer to the reference solution as the rotation speed is increased, whereas the Inxt model is accurate up to amplitudes of the order of one third of the beam length.

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