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DIRECT AND SPARSE CONSTRUCTION OF THE INVERSE OF THE CONSISTENT MASS MATRIX: GENERAL VARIATIONAL FORMULATION AND APPLICATION TO SELECTIVE MASS SCALING

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The primal goal of this contribution is the reduction of the computational costs of direct time integration methods, e.g. the central difference method, and asynchronous variational integrators (AVI) without substantial loss of accuracy. This is achieved via increase of the critical time step size with an alternative mass matrix formulation [1-4]. This formulation avoids the inversion of matrices on the global system level, i.e. an inverse of the mass matrix is directly assembled from the element matrices and this inverse yields a larger critical time step than the lumped mass matrix (LMM). It should be noted that a similar idea has been recently presented in [5]. This approach combines micro-inertia continua with additional length scales for mass scaling together with Neumann expansion for an approximate value of the inverse of a matrix. Here, a variational construction is followed.

The approach is based on the penalized Hamilton's principle [1] and biorthogonal shape functions [6]. The penalized Hamilton's principle uses independent variables for displacements, velocities and momenta and free penalty parameters. A weak form based on the principle is discretized in space using Bubnov-Galerkin finite element [1]. Biorthogonal shape functions for displacements and momenta allow the elimination of momentum parameters from equations without matrix inversion. The result of such a derivation is sparse (fill-in of CMM), consistent (preservation of translational inertia), stable (positive definite), accurate (the same convergence rate as for CMM or LMM) and symmetric inverse of the mass matrix, denoted as reciprocal mass matrix (RMM) herein. In order to increase the critical time step, the penalty term in the penalized Hamilton's principle is exploited, namely mass penalty between velocities and momenta is used, referred as variational mass scaling [1]. The shape functions for velocity are chosen to be element-wise constant, providing consistency and efficient scaling of the critical time step size. The local definition of velocity parameters enables their local elimination. This leads to a one-parametric family of variationally scaled reciprocal mass matrices (VSRMM).

The presented approach is validated for several eigenvalue and transient benchmarks for one-, two- and three-dimensional problems modelled with simplex finite elements. The lowest eigenvalues obtained with RMM and VSRMM provide the same accuracy as LMM or CMM. Speed-up for transient examples with the central difference method is in the range of 25-50%. The difference to results with LMM is below the engineering accuracy. It was also possible to reduce the dispersion error for a special choice of the penalty parameter, which is observed for an average of CMM and LMM. Reduction of the computational costs is also expected for transient time integration with AVI [7].

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