DIRECT AND SPARSE CONSTRUCTION OF THE INVERSE OF THE CONSISTENT
MASS MATRIX: GENERAL VARIATIONAL FORMULATION AND APPLICATION TO
SELECTIVE MASS SCALING

Anton Tkachuk*1 and Manfred Bischoff†

1 Institute of Structural Mechanics, Pfaffenwaldring 7, 70550, Stuttgart Germany
tkachuk,bischoff@ibb.uni-stuttgart.de, www.ibb.uni-stuttgart.de

Key Words: Selective Mass Scaling, Explicit Dynamics, Reciprocal Mass Matrix,
Parametrized Variational Principles.

The primal goal of this contribution is the reduction of the computational costs of direct time
integration methods, e.g. the central difference method, and asynchronous variational
integrators (AVI) without substantial loss of accuracy. This is achieved via increase of the
critical time step size with an alternative mass matrix formulation [1-4]. This formulation
avoids the inversion of matrices on the global system level, i.e. an inverse of the mass matrix
is directly assembled from the element matrices and this inverse yields a larger critical time
step than the lumped mass matrix (LMM). It should be noted that a similar idea has been
recently presented in [5]. This approach combines micro-inertia continua with additional
length scales for mass scaling together with Neumann expansion for an approximate value of
the inverse of a matrix. Here, a variational construction is followed.

The approach is based on the penalized Hamilton’s principle [1] and biorthogonal shape
functions [6]. The penalized Hamilton’s principle uses independent variables for
displacements, velocities and momenta and free penalty parameters. A weak form based on
the principle is discretized in space using Bubnov-Galerkin finite element [1]. Biorthogonal
shape functions for displacements and momenta allow the elimination of momentum
parameters from equations without matrix inversion. The result of such a derivation is sparse
(fill-in of CMM), consistent (preservation of translational inertia), stable (positive definite),
accurate (the same convergence rate as for CMM or LMM) and symmetric inverse of the
mass matrix, denoted as reciprocal mass matrix (RMM) herein. In order to increase the
critical time step, the penalty term in the penalized Hamilton’s principle is exploited, namely
mass penalty between velocities and momenta is used, referred as variational mass scaling [1].
The shape functions for velocity are chosen to be element-wise constant, providing
consistency and efficient scaling of the critical time step size. The local definition of velocity
parameters enables their local elimination. This leads to a one-parametric family of
variationally scaled reciprocal mass matrices (VSRMM).

The presented approach is validated for several eigenvalue and transient benchmarks for one-,
two- and three-dimensional problems modelled with simplex finite elements. The lowest
eigenvalues obtained with RMM and VSRMM provide the same accuracy as LMM or CMM.
Speed-up for transient examples with the central difference method is in the range of 25-50%.
The difference to results with LMM is below the engineering accuracy. It was also possible to
reduce the dispersion error for a special choice of the penalty parameter, which is observed for an average of CMM and LMM. Reduction of the computational costs is also expected for transient time integration with AVI [7].

The support of Baden-Wüttemberg Stiftung within a project “Reduction of numerical sensitivities in crash simulations on high performance computers (HPC-10)” and fruitful discussions of the topic and its extensions with Michael Ortiz are gratefully acknowledged.

REFERENCES