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A VMS THREE-FIELD STABILIZED FORMULATION FOR INCOMPRESSIBLE VISCOELASTIC FLUIDS

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In this paper, a new three-field VMS Finite Element stabilized formulation is proposed for the numerical solution of incompressible viscoelastic fluid flow that allows equal interpolation between the problem unknowns $\boldsymbol{\sigma}$-$\boldsymbol{u}$-p (deviatoric viscoelastic stress, velocity and pressure). The most important difference between the viscoelastic and the non-elastic fluids (Newtonian or non-Newtonian) is the time dependent nature of the hyperbolic constitutive equation that defines the fluid. The constitutive equation has to be simultaneously solved with the momentum and mass conservation equations. The three-field mixed problem analized can lead different types of numerical instabilities. On the one hand, the incompressibility and the dominant convection associated to the classical Navier-Stokes problem ($\boldsymbol{u}$-p) and, on the other hand, the hyperbolic and convective nature of the constitutive equation that, in addition, is highly non-linearly when the elasticity of the fluid is increased.

The three-field mixed formulation of the incompressible Navier-Stokes problem in both, the viscoelastic and the non-elastic case, must satisfy two inf-sup conditions, one between velocity-pressure and the other between velocity-stress [1]. These two restrictions reduce the choices of stable finite elements spaces that allow to discretize the unknowns (see [2]). The proposed stabilized formulation is framed in the context of sub-grid scale (SGS) method [3] and allows equal interpolation between the unknowns even in problems where convection is dominant or the elastic component is important in the fluid.

The work includes a description of the proposed method, some details of the treatment of the non-linearity terms presented in the constituive equation and a study of the convergence of the formulation for the different fields ($\boldsymbol{\sigma}$-$\boldsymbol{u}$-p). Finally, some results obtained in the classical 4:1 contraction benchmark for this type of fluids are shown.

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