

STABILISATION PARAMETER DETERMINATION FOR THE STOKES EQUATIONS

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Stabilisation parameters within the variational multiscale method for the Stokes equations are determined by two different methods. The linear[1] and nonlinear[2] models of stabilisation parameters are considered for a comparison. Firstly, optimal coefficients of stabilisation parameters are found using a goal-oriented model-constrained technique to seek the minimum of the L2 norm error. Solutions for different mesh sizes are analysed. It turns out that for both the linear and nonlinear models the optimal coefficient of the PSPG term tends to converge, and the one of the LSIC term decreases as the mesh becomes fine.

The coefficients of the stabilisation parameters, obtained through the goal-oriented approach, are contrasted with optimal coefficient values that are obtained by using the Variational Germano Identity[3, 4]. It is evident from the results that the performance of the Germano approach is very sensitive to the selected functional form of the stabilisation parameters. The coefficients obtained via the use of the Germano Identity follow a similar trend to those found via the goal-oriented approach as the mesh size is refined. Furthermore it is demonstrated that there is evidence that the Germano relations can potentially be used, a-priori, to assess the suitability of stabilisation parameters.

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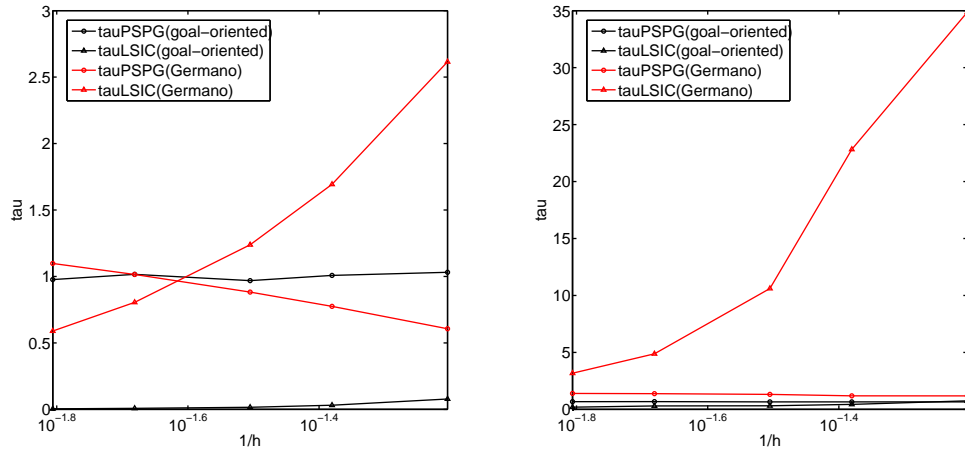


Figure 1: Variation of τ over the mesh size. Left: linear stabilisation parameters, right: nonlinear stabilisation parameters.

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