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LOCAL PROJECTION TYPE STABILISATION APPLIED TO INF-SUP STABLE DISCRETISATIONS OF THE OSEEN PROBLEM. PART I

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We consider the Oseen problem

$$\begin{aligned} -\nu\triangle u + \big(b\cdot\nabla\big)u + \sigma u + \nabla p &= f &&\text{in } \Omega,\\ \operatorname{div} u &= 0 &&\text{in } \Omega,\\ u &= 0 &&\text{on } \partial\Omega, \end{aligned}$$

where $\nu > 0$ and $\sigma \geq 0$ are constants and $b \in \big(W^{1,\infty}(\Omega)\big)^d$ with div $b = 0$ is a given velocity field. The Oseen problem can be considered as a linearisation of the steady ($\sigma = 0$) and the time-discretised non-steady ($\sigma > 0$) Navier–Stokes equations, respectively. In the latter case, one typically has $\sigma \sim 1/\triangle t$ with the time step length $\triangle t$ of an implicit time integration scheme while the stationary case corresponds to $\triangle t = \infty$.

Inf-sup stable finite element discretisations of the Oseen problem are considered. Hence, no additional pressure stabilisation is needed. However, the standard Galerkin method still suffers in general from global spurious oscillations in the velocity field which are caused by the dominating convection.

Local projection stabilisation methods will be used to overcome this difficulty. The stabilisation is based on a projection from the underlying approximation space onto a discontinuous projection space. Stabilisation is derived from additional weighted L^2-control on the fluctuation of the gradient of the velocity or only parts of it like divergence and/or derivative in streamline direction.

The convergence analysis for both the one-level and the two-level local projection stabilisation applied to inf-sup stable discretisations of the Oseen problem will be presented in a unified framework.

Two different stabilisation terms

$$S_h^a(u,v) := \sum_{M\in\mathcal{M}_h} \Big(\tau_M \big(\kappa_M^1 (b_M\cdot\nabla)u, \kappa_M^1 (b_M\cdot\nabla)v\big)_M + \gamma_M\big(\kappa_M^2 \mathrm{div}\ u, \kappa_M^2 \mathrm{div}\ v\big)_M\Big)$$

and

$$S_h^b(u,v) := \sum_{M\in\mathcal{M}_h} \mu_M\big(\kappa_M^3\nabla u, \kappa_M^3 \nabla v\big)_M$$

with non-negative coefficients τ_M, γ_M, μ_M are considered where $\kappa_M^i := \mathrm{id} - \pi_M^i$, $i = 1,2,3$, denote the fluctuation operators, $\pi_M^i : L^2(M) \to D^i(M)$ the local projection operators into finite dimensional spaces. Note that S_h^a introduces control over the fluctuations of the derivative in streamline direction and over the fluctuations of the divergence separately whereas S_h^b controls the fluctuations of the gradient.

We will consider known inf-sup stable pairs of finite element spaces which approximate the velocity components by elements of order r and the pressure by elements of order $r-1$. We will discuss conditions on the spaces and the stabilisation terms which ensure the convergence order r in an appropriate norm on the product space of velocity and pressure.

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