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GEOMETRICALLY-EXACT ISOGEOMETRIC FORMULATION FOR TWO-DIMENSIONAL, SLENDER, EULER-BERNOULLI BEAMS: STATIC AND DYNAMIC CONSIDERATIONS.

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Beam theory is a useful tool to model deformation of slender structures. In several instances, finite displacement is the goal of simulations for which linear beam theories based on infinitesimal strain cannot be used. The geometrically-exact beam formulation, first introduced by Reissner [1], takes into account large nonlinear deformations of bending, axial and shear-type and its finite-element (FE) formulation is well used nowadays. For slender structures such as cables or rods, shear deformations can be neglected leading to the Euler-Bernoulli assumption. This simplification eliminates the issue of shear-locking in numerical solutions and is rotation-free. Irschik and Gerstmayr [2], presented an interpretation of the strain measures and stress resultants for the geometrically-exact, Euler-Bernoulli formulation in terms of non-linear continuum mechanics. A linear relation between the second Piola-Kirchhoff stress and the Green-Lagrange strain, as given by the Saint Venant-Kirchhoff model, results in a nonlinear relationship between stress resultants and strain and curvature, thus introducing a nonlinear material model. On the other hand, a linear constitutive law between Biot Stress and Biot strain, as adopted in the current work, results in linear material behavior [2]. However even a linear constitutive law at the element level, the weak form of the problem, involves second-order derivatives for which a solution based on the Galerking method requires at least $\mathcal{C}^1$-continuous basis functions across the mesh.

Isogeometric analysis (IGA), first introduced by Hughes et al.[3] generalize the FE method by considering Non-Uniform, Rational, B-Spline (NURBS) shape functions for witch it is possible to increase the degree of continuity through k-refinement. For multi-patch, since the continuity is only $\mathcal{C}^0$ across the global geometry, possible solution is provided by the bending-strip method recently adapted to rotation-free, bending-stabilized cables

[4]. Note also that in [4], Saint Venant-Kirchhoff model with linear relation between the second Piola-Kirchhoff stress and the Green-Lagrange strain is used [2].

Analytical solutions for a set of beam problems, including a newly-derived solution for initially-curved beam, are derived based on the extensible-elastica and are used as reference to verify the convergence orders of the numerical errors under h-refinement using a-priori error estimates of high-order PDEs [5]. It is found that the convergence order is in agreement with the theory despite the fact that the error estimates are derived for linear problems. The formulation is rotation-free and is of great interest in dynamic, decreasing the number of degree of freedom, and lending to diagonalized mass matrices. Considering the generalized-α time scheme [3] advantages of this new formulation are showed through the example of wave propagation in multiple, post-buckled beam [6] and comparison with alternative shear-free formulations is done.

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