

# A NEW APPROACH FOR MODAL SYNTHESIS OF A VIBROACOUSTIC PROBLEM

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## 1 Introduction - Finite element method for vibroacoustics

This paper focuses on the prediction of the coupled vibroacoustic behaviour of a structure-cavity system: the recent works in the field of smart lightweight structures to reduce the structural vibrations and the radiated noise require to be able to precisely predict this vibroacoustic behaviour for an adequate design of control laws. A classical way to solve such a problem is to use the finite element method commonly adopted for industrial problems. The method based on the (U-displacement,P-pressure) formulation leads to solve the following system for an undamped problem [1],

$$\begin{bmatrix} M_s & 0 \\ \rho_f C^T & M_f \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} K_s & -C \\ 0 & K_f \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix} \quad (1)$$

where  $K_s$ ,  $M_s$  are the structural stiffness and mass matrices,  $K_f$ ,  $M_f$  are the matrices respectively corresponding to the discretization of kinematic energy and compressibility matrix of fluid,  $C$  is the coupling matrix,  $\rho_f$  denotes the fluid density and  $F_s$  is an external force vector. The direct resolution of this problem is time-consuming due to the large-scale finite element models. Component mode synthesis and modal reduction methods provide methods to reduce the number of degrees of freedom.

## 2 The Combined Approximation (CA) method for vibroacoustics

The Combined Approximation approach has been developed in the 1990's for reanalysis problems as involved in structural optimization studies [2]. The procedure is applied here to a coupled vibroacoustic problem by considering it as a modification of the uncoupled problem and writing the global system 1 on the  $v^{th}$  eigenvector  $\varphi_v$  as[3],

$$[(K_0 + \mu_K M_0) + (\Delta K + A) - (\omega^2 + \mu_M)(M_0 + \Delta M)] \varphi_v = 0 \quad (2)$$

$K_0 = \begin{bmatrix} K_s & 0 \\ 0 & K_f \end{bmatrix}$  and  $M_0 = \begin{bmatrix} M_s & 0 \\ 0 & M_f \end{bmatrix}$  are the uncoupled matrices,  $\Delta K = \begin{bmatrix} 0 & -C \\ 0 & 0 \end{bmatrix}$  and  $\Delta M = \begin{bmatrix} 0 & 0 \\ \rho_f C^T & 0 \end{bmatrix}$  denote the mass and stiffness perturbations due to the vibroacoustic coupling. A deflation scheme is used to avoid problems due to sparse matrices,  $\mu_K$  and  $\mu_M$  are arbitrary spectral shift scalars, and  $A$  is a matrix such that  $\mu_K = \mu_M, A = \mu_K \Delta M \quad \forall \mu_K \neq 0$  or  $\mu_M = 0, A = -\mu_K M_0 \quad \forall \mu_K \neq 0$ .  $\varphi_v$  is then computed using a Taylor series expansion as

$$\varphi_v = (I + B_0)^{-1} r_0 = (I - B_0 + B_0^2 - \dots) r_0 \quad (3)$$

where  $B_0 = (K_0 + \mu_K M_0)^{-1} (\Delta K + A)$  and  $r_0$  depends on the eigenvector  $\varphi_{0v}$  of the uncoupled problem,  $r_0 = (K_0 + \mu_K M_0)^{-1} (M_0 + \Delta M) \varphi_{0v}$ . Global matrices are then projected on the obtained projection basis and the eigenvalue reduced problem is solved.

### 3 Application

The method is applied to an air-filled parallelepipedic cavity with 5 rigid faces and an aluminium elastic plate on its upper face. Eigenfrequencies are compared to those given by Ansys software based on the frequency derivative method which uses an orthogonal set of Krylov sequence of vectors [4].

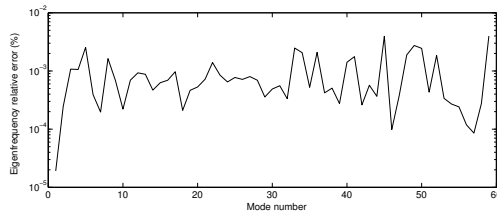


Figure 1: Eigenfrequency relative error (%) between Ansys results and the proposed approach based on the CA method

Results are very similar with less than 0.1% error. One main interest of the proposed approach relies on the reduced number of computation steps as it is not an iterative approach, but the results depend on the order development in the Taylor expansion. The method thus appears as an interesting alternative to compute a reduction basis for coupled problems and its application can undoubtedly be extended to nonlinear applications.

### REFERENCES

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