

SPACE-TIME FINITE ELEMENT ANALYSIS OF FREE-SURFACE FLOWS WITH REGARD TO WATER WHEELS

Hans-Henning Schippke¹, Christian Seidel¹ and Dieter Dinkler¹

¹ TU Braunschweig, Institut für Statik, Beethovenstr. 51, D-38106 Braunschweig

Key words: *Incompressible Navier-Stokes Equations, Free-Surface Flow, Space-Time Finite Element Method, Moving Boundary and Interface, Level-Set.*

Due to the transformation process currently taking place in the energy production sector, the use of energy gained from renewable power sources is increasing. In case of deriving energy from running water hydropower turbines are the state-of-the-art machinery, whereas water wheels are the method of choice in order to convert the potential energy as well.

The rotating water wheel, surrounded by air and water, poses a three field system consisting of a structure and two fluids. Due to the elasticity of the rotor blades the stresses of the fluid acting onto the structure deform the blades, which in return yields a time-dependent flow domain. Therefore water wheels in a streaming fluid represent a typical example of fluid-structure interaction. In particular interaction phenomena with a significant influence on the performance on the water wheel take place in the inflow and outflow region as well as in between the rotor blades in the transition region from head to tail water level. The interaction phenomena occurring there do highly influence the efficiency of the overall system.

In this contribution the governing equations of incompressible fluid flow are presented using primal variables velocity and pressure and are discretised via the space-time finite element method [1]. The discretised model equations of the fluid are stabilised using an SUPG/PSPG approach. Shape and test functions are continuous within the space-time slabs, while across the space- time slabs the shape and test functions are continuous only in space, but discontinuous in time yielding a time-discontinuous Galerkin approach.

Due to the moving rotor blades a mesh moving technique needs to be incorporated into the computational set-up. Considering the occurring large but regular displacements of the flow boundary arising from the rotating rotor blades the shear-slip mesh update method (SSMUM) [2] as discontinuous mesh moving technique is applied.

In this contribution the interface between air and water is described implicitly only using a level set method [3] with a signed distance function. In doing so, the original three field system reduces to a two field system in which the interface between the remaining fluid with varying material parameters and the structure is discretised explicitly via coincident nodal points in combination with the SSMUM.

By considering different test examples the accuracy and robustness of the numerical method developed is examined. In particular, the order of integration of the jump term of the shear-slip layer elements and mass conservation in the two fluid phases as well as energy conservation of the whole system will be discussed in detail. Finally, the verified numerical method is applied to the overall water wheel system and the gained numerical results are compared to experimental data.

REFERENCES

- [1] Tezduyar, T. E.; Behr, M.; Liou, J.: A new strategy for finite element computations involving moving boundaries and interfaces - The deforming-spatial-domain/space-time procedure: 1. The concept and the preliminary numerical tests. *Computer Methods in Applied Mechanics and Engineering*, **94**, 339 - 351, 1992.
- [2] Behr, M.; Tezduyar, T. E.: The Shear-Slip Mesh Update Method. *Computer Methods in Applied Mechanics and Engineering*, **174**, 261 - 274, 1999.
- [3] Osher, S.; Sethian, J. A.: Fronts Propagating with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations. *Journal of Computational Physics*, **79**, 12 - 49, 1988.