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EXTREMELY HIGH-ORDER MULTIOPERATORS-BASED SCHEMES FOR SMOOTH AND DISCONTINUES FLUID DYNAMICS SOLUTIONS

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The multioperators idea first proposed in [1] and described in particular in [2], [3] consists of using linear combinations of the form

$$L_M(s_1, s_2, \ldots, s_M) = \sum_{i=1}^{M} \gamma_i L_m(s_i)$$

where $L_m(s)$ is one-parameter family of m-th-order compact approximation on uniform mesh with mesh size h to target linear operator L defining some numerical analysis formula. Coefficients γ_i are the solutions of the linear systems resulting from annihilation of low-order terms in the truncation errors for $L_M(s_i)$. In this way, one can construct arbitrary high order discretization by choosing arbitrary large sets of M values of parameters s. Operator L_M was labelled [1] by multioperator while $L_m(s_i), i = 1, 2, \ldots M$ can be viewed as basis operators. Calculations of multioperators actions on known grid functions can be performed in a parallel manner by calculating simultaneously actions of their basis operators.

In the case of first derivatives ($L = \partial/\partial x$)), multioperators can provide at least $O(h^{m+M-1})$ truncation errors. By choosing properly s_i, one can construct upwind – downwind multioperators pairs (that is pairs of multioperators with the same skew-symmetric components and positive – negative self-adjoint components). The self-adjoint components play role of a high-order built-in filter of spurious oscillations acting only on the shortest waves supported by meshes. The dispersion –dissipation properties of $L_M(s_i)$ can be controlled by using s_i as free parameters allowing one to optimize multioperators-based schemes.

In the talk, the basis operators and resulting multioperators are discussed, several types of conservative schemes with multioperators for CFD applications being outlined. It is shown in particular that the optimized schemes can provide spectral-like resolution.

Upon briefly describing the theory, the following items will be presented.

(1) Testing against exact solutions of the schemes based on the 10th- and 18th-order multioperators which shows extremely high accuracy and high resolution.

(2) Results of high fidelity Navier-Stokes calculations of instability of hot subsonic jets with noise radiation using the 10th-order multioperators approximations.

(3) 2D and 3D Navier-Stokes calculations of instability of underexpanded supersonic jets with 7th-9-th order multioperators showing very clear pictures of the screech effect.

(4) The first results of using hybrid schemes with blending 9th-order multioperators and a low order monotone scheme to deal with strong discontinuities. The idea is to provide reasonable description of solutions at shocks and contacts vicinities with high order convergence away from them. The results of testing against extremal Riemann problems (the Mach numbers are up to 1000) show that hybrid multioperators schemes can be quite competitive.

In cases (2) and (3), condensing of grid points in high gradients regions was performed via smooth mappings into computational domains.

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