11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

Temporal Error Estimation and Adaptive Time Step Control in Unsteady Flow Simulations

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Key Words: *Unsteady Flow, Time Accurate, Runge-Kutta, High Order, Adaptive Time Stepping, Error Estimation*

An important factor to control the accuracy of unsteady simulations is the estimation of a numerical scheme's temporal errors. This error estimation can be used to control the time step size, which in turn is beneficial either for achieving a simulation with a prescribed (high) accuracy or for reducing the simulation time by increasing the time step size. The latter procedure is of special interest, since it allows considerable simulation acceleration at the same accuracy as a method with fixed time step size. Furthermore, the adaptive time stepping technique may be considered as a more user-friendly method, since the user does not have to estimate the time step size a-priori and the numerical error a-posteriori, but simply prescribes the error tolerance, and the method automatically adapts to this prescription.

In this paper several **E**xplicit first stage, **S**tiffly accurate **D**iagonally **I**mplicit **R**unge-**K**utta (ESDIRK) schemes [1]-[3] for the numerical integration of the compressible Navier-Stokes equations are considered. This type of numerical schemes allows for embedded error estimation using two methods with different order of accuracy. Based on this error estimation an adaptive time stepping is done using different controller types.

For the basic validation of the embedded error estimation the test problem "E4" of Enright [4] is used. This is a stiff non-linear system of ordinary-differential equations, which is representative of fluid flow with large unsteady gradients. An analytical solution is available for this test case, which allows an exact comparison to the numerical solution. The results confirm the order of accuracy of the investigated methods and their embedded schemes (Figure 1). For the estimated errors a wide spread is observed, some methods show a very good estimation of the error (Figure 2). The aforementioned time step controllers are tested using this academic problem.

Following the basic investigation of the ESDIRK methods using the E4 problem, the characteristics and accuracy of the methods in the context of the compressible Reynolds-Averaged Navier-Stokes flow solver TRACE [5] are then studied. The first problem investigated is the advection of a localized entropy disturbance in an otherwise uniform subsonic flow. In the second problem laminar and turbulent flows around a circular cylinder are considered. In the final paper a systematic study of the implicit Runge-Kutta time stepping schemes with adaptive time step control will be presented along with a detailed analysis of the error estimators and the efficiency of the time-step adaption algorithms.

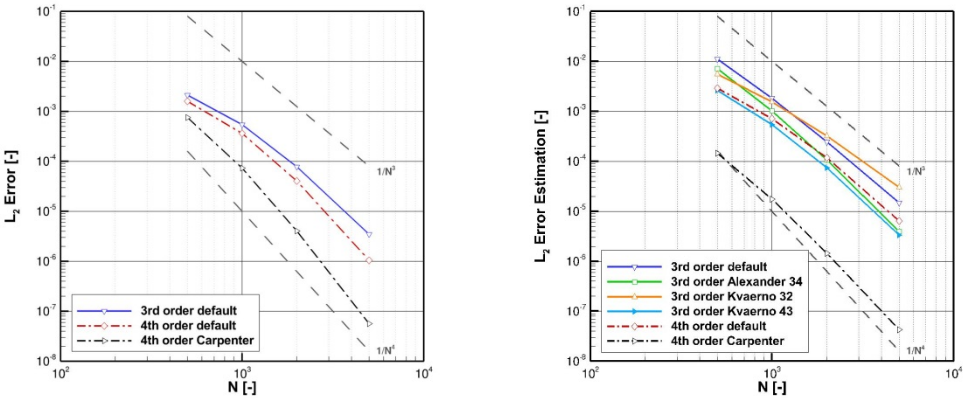

Figure 1: L2 Error of different ESDIRK schemes compared to analytical solution (left), L2 error of error estimator, each time step initialized with analytical solution (right).

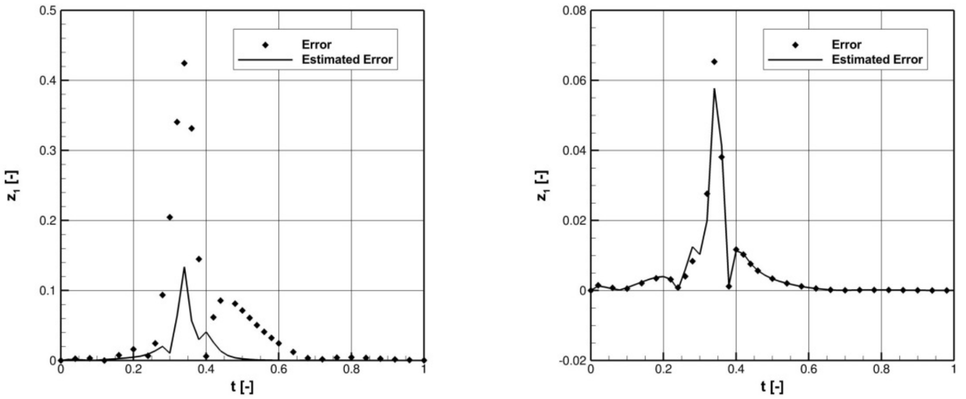

Figure 2: Comparison of analytical and estimated error for methods of Alexander [1] ESDIRK(3,2) (left) and ESDIRK(3,4) (right)

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