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A NEW ANALYTIC PRECONDITIONER FOR THE ITERATIVE SOLUTION OF DIRICHLET EXTERIOR SCATTERING PROBLEMS IN 3D ELASTICITY

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Motivations. When considering the solution of scattering problems of time-harmonic elastic waves by a three-dimensional rigid obstacle, the main difficulty in the numerical simulation comes from the unbounded characteristic of the computational domain. The boundary element method (BEM) is one possible approach to overcome this issue. The method results from the discretization of boundary integral equations (BIE). In traditional boundary element (BE) implementations, the dimensional advantage with respect to domain discretization methods is offset by the fully-populated nature of the BEM coefficient matrix. The Fast Multipole Method (FMM) permits to overcome the drawback of the fully-populated matrix by introducing a fast and approximate method to compute the linear integral operator in conjunction with the use of an iterative solver (e.g. GMRES). In 3D elastodynamics the FM-BEM has been shown to be efficient [1] with solution times of order $O(N \log N)$ per iteration (where N is the number of BE degrees of freedom). However, the number of iterations in GMRES can significantly hinder the overall efficiency of the FM-BEM even though an algebraic preconditioner is applied [2]. Preconditioning the FM-BEM is therefore an important practical issue. A possible approach consists in exploiting mathematical properties of the relevant continuous integral operators.

Methodology. In [3], Darbas *et al.* present the successful combination of an On-Surface Radiation Condition (OSRC)-based preconditioner and a FM-BEM to define an efficient solver for 3-D acoustic scattering by sound-hard obstacles at high frequencies. The idea is to consider a judicious integral representation of the scattered field which naturally incorporates a regularizing operator. The objective is to force the boundary integral operator arising from this representation to become a compact perturbation of the identity operator. When considering Dirichlet boundary value problems, the regularizing operator is a high-frequency approximation to the Dirichlet-to-Neumann (DtN) operator, and is constructed in the framework of the OSRC method. This approximate method has been successfully proposed and applied in acoustics and electromagnetism but its extension to 3D elastodynamics is involved. A preparatory theoretical work has been proposed by Darbas and Le Louër [4] to derive an OSRC-like preconditioner to solve Dirichlet exterior scattering problems in 3D elasticity.

Results. We propose here a first extension of the OSRC method to 3D elastodynamic problems and its application as a preconditioner of the FM-BEM. This communication is organized as follows. First, the approximation of the DtN proposed in [4] and its application in the OSRC context is presented. The low costs of the ORSC method to construct an approximate solution of the exterior Navier problem at high frequencies is demonstrated for different obstacles. Then, it is shown how to use this approximation of the DtN to precondition the FM-BEM. In particular, since this preconditioner is not intrusive, the two methods are used in a black box framework. Finally, the numerical efficiency of the combination of this OSRC-based preconditioner with a FM-BEM solver is presented on high-frequency 3-D cases. The independence of the iteration counts with respect to the mesh density and frequency is confirmed on numerical examples. The additional computational cost of the preconditioner is shown to be negligible compared to the cost of a FMM accelerated matrix-vector product.

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