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A TEMPORAL INTEGRATOR BASED ON SERIES RESOMMATION. APPLICATIONS TO FLUID-STRUCTURE INTERACTION

Aziz Hamdouni[1], Dina Razafindralandy[2] and Ahmad Deeb[3]

Laboratoire des Sciences de l'Ingénieur pour l'Environnement (LaSIE)
Université de La Rochelle
Avenue Michel Crépeau, 17042 La Rochelle Cedex, France
[1] ahamdoun@univ-lr.fr
[2] drazafin@univ-lr.fr
[3] ahmad.deeb@univ-lr.fr

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Most of classical algorithms rely on discretization methods to numerically integrate evolution problems. As a consequence, they generally necessitate a huge CPU-time since stability requirements impose a very small time step. So, despite the constant progress in computer science (power of computers, parallelization, . . .), engineers are still in quest of fast algorithms. It is with that in mind that we propose a new algorithm for time integration of ordinary or partial differential equations, which is well suited to computation in a large time interval.

Consider an ordinary or a semi-descretzied partial differential equation

$$\frac{\mathrm{d}\,u}{\mathrm{d}\,t} = A(u), \qquad\qquad u(t=0) = u_0. \tag{1}$$

The solution $u(t)$ is decomposed into a formal power series

$$\hat{u}(t) = \sum_{k=0}^{+\infty} u_k t^k. \tag{2}$$

The terms $u_k,\ k \geq 1$, are determined from a recurrence relation. For example, with the Navier-Stokes equations, the terms of the velocity and pressure series verify:

$$(k+1)u_{k+1} + \frac{1}{\rho}\nabla p_k = -\sum_{l=0}^{k}(u_l\cdot\nabla)u_{k-l} + \nu\Delta u_k, \qquad \operatorname{div} u_{k+1} = 0. \tag{3}$$

One has to solve a cascade of linear (Darcy) equations, which is much less time-consumming than the Navier-Stokes equations. Only the right-hand side changes at each order.

The numerical radius of convergence of the series $\hat{u}$ (and $\hat{p}$ for the Navier-Stokes equations) may be small, or even zero. In this case, a resummation procedure must be applied. The one chosen here is the Borel-Laplace resummation method, which enables to find a (sectorial) analytical solution which is asymptotic to the series. In the convergent case, this procedure prolonges the series outside its the convergence domain and may be considered as a convergence acceleration if the convergence radius is infinity.

The Borel-Laplace resummation method applies only if $\hat{u}$ is a Gevrey series, that is, $|u_k| < CA^k t^k k!$ for some constants C and A. Costin *et al.* showed in [1] that this condition is fulfilled with the Navier-Stokes equations. The resummation procedure can be summarized in three steps.

- First, one computes the Borel transforms of the series, $\mathcal{B}\hat{u}(\xi) = \sum_{k=0}^{+\infty} \frac{u_{k+1}}{k!}\xi^k$, which is analytic at the origin.
- Next, $\mathcal{B}\hat{u}(\xi)$ is prolonged analytically into a function $P(\xi)$ along a semi-line d linking the origin to the infinity. Numerically, this step may be realized with Padé approximants.
- Finally, the Borel sum $\mathcal{S}(t)$ is the Laplace transform of $P(\xi)$ along the line d, added to u_0:

$$\mathcal{S}(t) = u_0 + \int_d P(\xi)\,\mathrm{e}^{-\xi/t}\,\mathrm{d}\,\xi.$$

$\mathcal{S}(t)$ is taken as an approximation of $u(t)$ as long as t belongs to domain of validity. The method is iterated to reach higher values of t.

The above algorithm has been successfully used in [2] to solve some model ODE's and PDE's. It has been shown that this time integration method much less iteration numbers than other classical explicit ones. In the present communication, we are interested in applications in engineering science. The performance of this time integration scheme for fluid flow simulation and fluid structure interaction problems will be presented.

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