11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

IMPLEMENTATION OF THE Sℓ_1QP METHOD, AND ITS APPLICATION TO OPTIMIZATION OF A CASCADE AIRFOIL SHAPE

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Key words: *Optimization, SQP, Trust Region, Penalty, Cascade Airfoil, Design.*

Sequential Quadratic Programming (SQP) methods are effective for solving a nonlinear constrained optimization problem

$$\begin{aligned} min\ f(\mathbf{x}) & & & \\ subject\ to\ c_i(\mathbf{x}) &= 0, & i &\in \mathcal{E} \\ c_i(\mathbf{x}) &\geq 0, & i &\in \mathcal{I} \end{aligned}$$

SQP methods linearlize both the inequality and equality constraints to obtain the quadratic program (QP) subproblem, and are used both in line search tust-region frameworks [1]. Trust-region SQP methods add a trust region constraint to the subproblem. The Sℓ_1QP method transforms this subproblem into another form; it moves the linearlized constraints into the objective function of the quadratic program in the form of an ℓ_1 penalty term [1, 2]. We have implemented the Sℓ_1QP method.

Preliminary numerical tests are done for the implemented Sℓ_1QP method. The problem

$$\begin{aligned} min\ -x_1 + 10x_1^2 + 10x_2^2 & & \\ subject\ to\ x_1^2 + x_2^2 - 1 &= 0 \\ x_1 - 0.5 &\geq 0 \\ x_2 + 0.5 &\geq 0 \end{aligned}$$

is solved staring from different initial guesses. Table 1 compares the results of the Sℓ_1QP, line search SQP [1] and NLPQLP [3] methods. The line search SQP and NLPQLP methods utilize monotone and non-monotone line search, respectively.

Then, the Sℓ_1QP method is applied to optimization of a cascade airfoil shape. The optimization is formulated as a nonlinear constrained optimization problem. The objective function is defined which shows an aerodynamic characteristic of the airfoil, and is minimized subject to the constraints. The Sℓ_1QP method requires the gradient of the objective

Table 1: Results of $S\ell_1QP$, line search SQP, and NLPQLP methods (nit: number of major iterations, nqo: number of qp problems solved)

Case	initial guess		$S\ell_1QP$		SQP		NLPQLP	
	x_1	x_2	nit	nqp	nit	nqp	nit	nqp
A	1.0	0.5	4	4	4	6	6	6
B	0.5	0.5	7	7	4	7	10	10
C	-1.5	-1.5	10	10	9	15	—	—
D	3.0	3.0	8	8	85	167	9	9
E	10	10	15	15	55	106	10	10

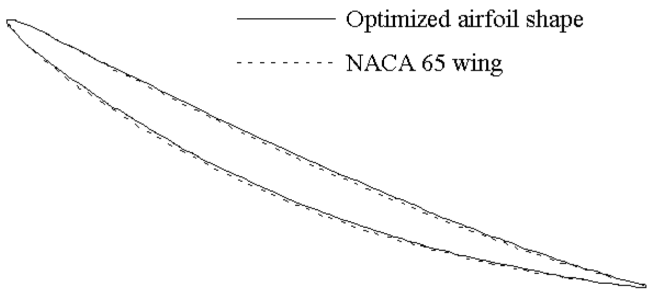

Figure 1: NACA 65 wing (dotted line) and optimized airfoil shape (solid line).

function with respect to the design variables at each iteration. The gradient is computed by applying the implicit function theorem to the discretized Navier-Stokes equations [4]. A numerical experiment is presented for optimization of a compressor cascade shape. NACA 65 wing is chosen to be the initial airfoil shape. Figure 1 shows the initial shape and the optimized shape. The total pressure loss decreases about 27 % for the optimized shape.

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