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HOMOTOPY FROM PLANE COUETTE FLOW TO PIPE FLOW

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Invariant sets of Navier-Stokes equations, called *exact coherent structures*, play important roles in dynamical systems approaches for understanding subcritical transition from laminar state to turbulence. Two canonical shear flows which are linearly stable for all Reynolds numbers and therefore undergo subcritical transition, *i.e.* plane Couette flow (PCF) and pipe flow (PF), are particularly important. Several exact coherent structures for both flows are known to date (see [1] and references therein) and it would be of interest to examine connections among them in terms of symmetry. In spite of the fact that both PCF and PF are very different geometrically, we could take annular Poiseuille-Couette flow as an intermediary, since it recovers PCF by taking the narrow gap limit and also PF by taking the limit of vanishing inner cylinder. This is the flow we consider in the present report.

Annular Poiseuille-Couette flow is an incompressible viscous fluid motion between two infinitely long concentric cylinders, driven by an axial pressure gradient as well as a steady axial motion of the inner cylinder relative to the static outer cylinder. This system is governed by R, the Reynolds number induced by the sliding motion of the inner cylinder, and R_p, the Reynolds number induced by the axial pressure gradient. In particular, the case with $R_p = 0$ is called sliding Couette flow (SCF).

Disturbances superimposed on the basic state are expanded by truncated sums of the modified Chebyshev polynomials in the wall normal direction as well as the Fourier series in the axial and azimuthal directions. The partial differential equations for the disturbance are discretized by using the standard Chebyshev-collocation and Fourier-Galerkin techniques. We solve the resultant quadratic algebraic equations for the spectral coefficients by the Newton-Raphson iteration method.

Recently, a homotopy from PCF to SCF was caried out[1], where the steady three-dimensional PCF solution, usually referred to as *Nagata solution*[2], was taken as a seed and three-dimensional solutions of SCF were obtained through transformation from plane geometry. The solution branch thus continued to SCF with a finite radius ratio was classified as class-$\mathcal{P}$. Also found in [1] were two branches of mirror-symmetric solutions, classified as the class-$\mathcal{M}$, one of which bifurcates from the class-$\mathcal{P}$ solution. Although the two class-$\mathcal{M}$ branches belong to the same symmetry group, their spatial structures were different: one of them had large vortices whose scale was comparable to the gap size, whereas the other had double-layered vortices with half gap size. We shall show that the double-layered class-$\mathcal{M}$ solutions can be traced back to the mirror-symmetric solutions in PCF[3].

Our homotopy from SCF to PF takes three steps. (i) We add an axial pressure gradient to SCF, so that the basic velocity profile between the two cylinders coincides with this range of PF profile. Then, (ii) we consider the limit as the inner cylinder radius shrinks to zero. Even the infinitesimally slender inner cylinder has a non-negligible effect on the disturbance due to no-slip condition. To remove this effect, (iii) we carry out a homotopy to change of the basis functions gradually from no-slip to analytic at the centre.

We find that only the double-layered solution successfully reaches the PF limit. Comparison of the minimum bulk Reynolds number and the flow structures assures that our double-layered class-$\mathcal{M}$ solution is identical to M1 found by [4]. The single-layered class-$\mathcal{M}$ solution fails to continue to PF, experiencing a turning point in the step (i) before the velocity profile becomes as parabolic as PF for any R, although it is known that PF has a single-layered solution family labeled by N in [5]. The branch of class-$\mathcal{P}$ solution of SCF is unable to reach PF limit either, encountering many folds along the homotopy path when the pressure gradient increases.

In conclusion, we find that, among different symmetry classes of nonlinear solutions in sliding Couette flow, only the one with a double-layered vortex structure is successfully continued to both plane Couette flow and pipe flow, suggesting its robustness in transition.

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